Consistent quality control in ECG compression by means of direct metrics

Carlos Hernando–Ramiro[1], Manuel Blanco–Velasco[1], Lisandro Lovisolo[2] and Fernando Cruz–Roldán[1]

[1] Department of Teoría de la Señal y Comunicaciones, Universidad de Alcalá, Campus Universitario, 28805 Alcalá de Henares, Madrid, Spain (phone: +34 91 885 6708, fax: +34 91 885 6699).

[2] PROSAICO-DETEL/PEL, State University of Rio de Janeiro, Rio de Janeiro, Brazil.

E-mail: c.hernando@edu.uah.es, manuel.blanco@uah.es, lisandro@uerj.br, fernando.cruz@uah.es

June 2015

Abstract. The aim of electrocardiogram (ECG) compression is to reduce the amount of data as much as possible while the significant information for diagnosis is preserved. Objective metrics that are derived directly from the signal are suitable to control the quality of the compressed ECGs in practical applications. Many approaches have employed figures of merit based on the percentage root–mean–square difference (PRD) for this purpose. The benefits and drawbacks of the PRD measures along with other metrics for quality assessment in ECG compression are analysed in this work. We propose the use of the root mean square error (RMSE) for quality control, because it provides a clearer and more stable idea about how much the retrieved ECG waveform, which is the reference signal to establish diagnosis, separates from the original. For this reason, the RMSE is applied here as target metric in a thresholding algorithm that relies on the retained energy. A state–of–the–art compressor based on this approach and its PRD–based counterpart are implemented to test the actual capabilities of the proposed technique. Both compression schemes are employed in several experiments with the whole MIT–BIH Arrhythmia Database to assess both global and local signal distortion. The results show that, using the RMSE for quality control, the distortion of the reconstructed signal is better controlled without reducing the compression ratio.

PACS numbers: 87.85.Ng Submitted to: *Physiol. Meas.*

Keywords: direct metrics, ECG compression, Percentage Root–mean–square Difference (PRD), quality control, retained–energy thresholding, Root Mean Square Error (RMSE), subband coding.

1. Introduction

The electrocardiogram (ECG) is a biological signal that represents the electrical activity of the heart. The ECG recording conveys important information for the analysis and evaluation of the patient's health condition related to the cardiac system. Nowadays, remote applications such as ECG monitoring, database storing and diagnosis are in widespread use due to the advance of the Internet and mobile networks (Engin, Caglav & Engin 2005, Iliev, Krasteva & Tabakov 2007, Capua, Meduri & Morello 2010). In this context, the utilization of ECG compression for telemedicine is among the most challenging, versatile, and interesting applications, in which data reduction is needed to increase the transmission rate, allowing thus the reception of high resolution signals in real–time.

In lossy ECG compression, part of the original information content is lost, so the reconstructed signal is not fully identical to the original. These techniques are under constant research because they achieve high levels of compression in comparison with lossless compression methods (Sayood 2000). Lossy ECG compressors can be classified into three categories (Blanco–Velasco, Cruz–Roldán, López, Ángel M. Bravo & Martínez 2004): direct methods, transform methods, and other compression methods. The first group has provided good results in the past, but, in the recent years, most of the contributions fall into the last two categories. Regarding the second group, several different compression schemes have been developed (Chen & Itoh 1998, Alesanco, Olmos, Istepanian & García 2006, Chen, Ma, Zhang & Shi 2006, Benzid, Marir & Bougechal 2007, Benzid, Messaoudi & Boussaad 2008, Blanco–Velasco, Cruz–Roldán, Godino–Llorente & Barner 2007, Abo–Zahhad, Al–Ajlouni, Ahmed & Schilling 2013). Finally, from the last category, those techniques that rely on parameter extraction must be pointed out (Zigel, Cohen & Katz 2000a, Ouamri & Nat-Ali 2007, dos Santos Guimaraes, Lovisolo, Blanco–Velasco & Cruz–Roldán 2010) as well as the methods based on nearly–perfect reconstruction filter banks (Blanco–Velasco, Cruz–Roldán, López, Ángel M. Bravo & Martínez 2004, Blanco–Velasco, Cruz–Roldán, Godino–Llorente & Barner 2004, Blanco–Velasco, Cruz–Roldán, Moreno–Martínez, Godino–Llorente & Barner 2008).

Increasing the Compression Ratio (CR) is the main objective in lossy compression, but, due to distortion, the signal quality is an issue that should always be kept under control. Compressors must not only reduce the data to be transmitted but, at the same time, they must assure that the reconstructed signal is similar enough to the original one. For this purpose, the error that is introduced must be constantly monitored and constrained during the compression procedure. In order to achieve an objective and automatic assessment, one can distinguish between direct and indirect metrics. Direct metrics are calculated in a straightforward way from the samples of the signal, without the necessity of obtaining any other ancillary parameters. For this reason, direct metrics comprise mainly conventional quality measures that do not depend on the type of signal that is assessed. On the other hand, indirect metrics rely on additional inputs, apart

from the original samples, that are usually related to features of the signal with some clinical meaning. In the case of the ECG, these parameters could be the duration, shape or peak value of its different waves (Zigel, Cohen & Katz 2000b). In general, indirect metrics tend to be seen as more accurate, as their analysis covers a wide range of characteristics that are important in clinical diagnosis. However, their complexity makes them inappropriate for real–life applications that cannot afford the time or the resources required for the computation of additional parameters.

On the other hand, the quality of the reconstructed signal can also be tightly controlled with direct metrics. Since clinical analysis of the ECG is carried out through visual inspection, an appropriate quality control implies that the diagnosis attained with either the original or the reconstructed signal has to be the same. As a result, the quality is understood in terms of preserving the waveform. This criteria can be met by means of consistent direct metrics that are able to restrict the error equally in any part of the signal. One of the direct metrics that are most commonly used to assess quality is the Percentage Root–mean–square Difference (PRD). In this sense, both the CR and the PRD are the main issues considered in ECG compression (Chen & Itoh 1998, Blanco–Velasco, Cruz–Roldán, Godino–Llorente & Barner 2004, Blanco–Velasco, Cruz–Roldán, Godino–Llorente, Blanco–Velasco, Armiens–Aparicio & López 2005, Benzid et al. 2007, Blanco–Velasco et al. 2007, Benzid et al. 2008, Hung, Tsai, Ku & Wang 2009, Ku, Hung, Wu & Wang 2010, Aggarwal & Patterh 2012, Abo–Zahhad et al. 2013). Although these methods are able to constrain the PRD to a predetermined interval, this methodology by itself offers doubts about its suitability to assess the clinical validity of the resulting compressed signals. One reason is that the PRD does not provide meaningful information about the degradation of the signal. This fact has already been observed in (Alesanco, García, Serrano, Ramos & Istepanian 2006), where the advantages and drawbacks of using the PRD and the Root Mean Square Error (RMSE) to guarantee quality in ECG compression are discussed, and also in (Bazán-Prieto, Blanco-Velasco, Cárdenas-Barrera & Cruz-Roldán 2012a), in which the RMSE is proposed as a better suited objective parameter to measure quality in electroencephalographic signals.

In this work, direct metrics such as the PRD and the RMSE are studied to see which one offers a better option to assess properly the quality in ECG compression. A description and mathematical analysis of both types of measures provides a first idea about their consistency and stability. From this study, a quality control technique based on the retained energy, where the RMSE is proposed as target metric, is presented and included in a compression scheme based on thresholding. In order to prove also its practical application, a specific compressor is implemented with the addition of a state–of–the–art encoder. Thus, we complete an easy to use and low computational complexity system that can provide high–quality and good compression operating directly over raw ECGs, i.e., data directly supplied by the acquisition system, without any processing that may be the source of additional distortion. The proposed compressor is tested along with its PRD–based counterpart in order to evaluate its actual quality

control capabilities. The overall and local errors as well as the compression ratio are taken into account to provide evidence about the suitability of the RMSE to control the signal quality. In this way, it is demonstrated, through a representative set of experiments, that the use of the RMSE gives rise to a better performance in terms of quality control than the PRD, without degrading the compression performance.

This paper is organized as follows. In section 2, the direct metrics to be studied are introduced. Subsequently, the proposed compressors are presented in section 3. The simulations, including both quality and compression experiments, are developed in section 4. Finally, the conclusions drawn from the work are outlined in section 5.

2. Direct quality metrics

In multiple ECG compression algorithms, checking whether the reconstructed waveform has sufficient similitude with the original one by means of a direct metric is the way to control the signal quality. In most of these cases, the quality is continuously verified in real–time using the PRD as the figure of merit to guarantee an *a priori* defined quality target, also given in terms of the PRD. In (Blanco–Velasco, Cruz–Roldán, Godino–Llorente & Barner 2004, Blanco–Velasco et al. 2007), this idea was developed using wavelets and filter banks, respectively. Several other works have reported comparable approaches (Chen & Itoh 1998, Benzid et al. 2007, Benzid et al. 2008, Hung et al. 2009, Ku et al. 2010, Aggarwal & Patterh 2012, Abo–Zahhad et al. 2013) also based on the PRD as target.

Let $\mathbf{x}_N(n) = [x(n), x(n-1), \cdots, x(n-N+1)]^T$ be an N–samples vector representing a segment of the original ECG to be compressed. The distortion that affects the reconstructed ECG $\hat{\mathbf{x}}_N(n)$ can be examined through the coding error signal $\mathbf{e}_N(n) = \mathbf{x}_N(n) - \hat{\mathbf{x}}_N(n)$. The PRD is then defined as:

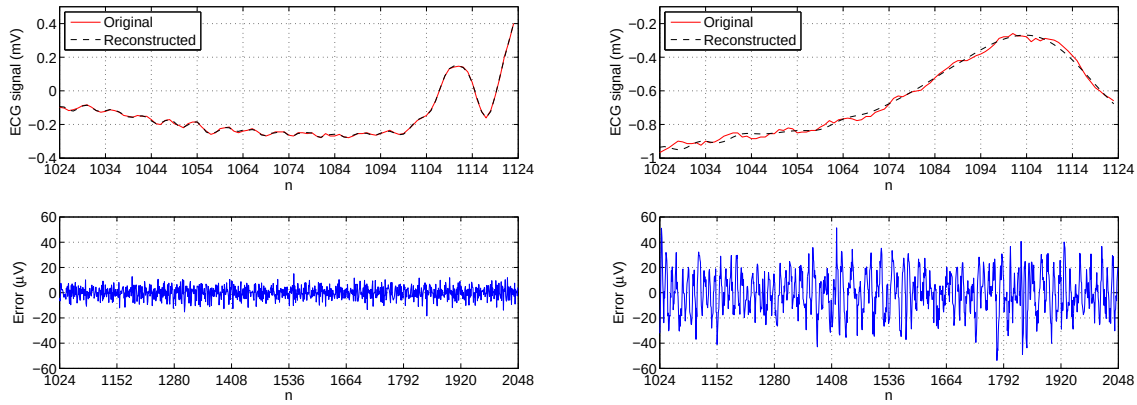
$$\mathcal{D}_N(n) = \sqrt{\frac{\sum_{l=0}^{N-1} e^2(n-l)}{\sum_{l=0}^{N-1} x^2(n-l)}} \cdot 100 = \frac{\|\mathbf{e}_N(n)\|}{\|\mathbf{x}_N(n)\|} \cdot 100, \tag{1}$$

where $\|\cdot\|$ denotes the Euclidean norm of the vector, referred to as its magnitude. The subscript N stands for the vector dimension while n represents the time lag at which the segment signal is taken. The PRD for $\mathbf{x}_N(n)$ is a scalar, $\mathcal{D}_N(n)$, that reports a relative measure of the error with respect to the signal magnitude $\|\mathbf{x}_N(n)\|$. In processing long–term ECGs, the compressor works over non–overlap short–term blocks:

$$\mathbf{x}_i(n) = \mathbf{x}_N(n + iN), \tag{2}$$

where $i = 0, 1, \cdots, K-1$, and K stands for the total number of signal segments that comprises the entire signal. After compression, a distortion figure is reported for each segment, $\mathcal{D}_N(n+iN)$, and the aim is to set this figure as close as possible to the target PRD:

$$\mathcal{D}_N(n + iN) \simeq \mathrm{PRD}_{target}. \tag{3}$$



(a) Segment of signal 111 with $x_{rms,1024}(2047) = 0.25$ mV and $\mathcal{D}_{1024}(2047) = 2.04\%$.

(b) Segment of signal 117 with $x_{rms,1024}(2047) = 0.82$ mV and $\mathcal{D}_{1024}(2047) = 2.02\%$.

Figure 1. Compression example of an ECG segment of 1024 samples from distinct signals of the MIT–BIH Arrhythmia Database. The upper panel of both figures shows a section of 100 samples from the original and the reconstructed signal superimposed. The compression error is depicted in the lower panel.

Thus, when evaluating the metric over the full signal, $\mathbf{x}_{KN}(n + (K-1)N)$, we ensure meeting the specification:

$$\mathcal{D}_{KN}(n + (K-1)N) \simeq \mathrm{PRD}_{target}, \tag{4}$$

where the subscript KN stands for vector length equal to $K \cdot N$ samples.

As the PRD depends on the signal amplitude, in consequence, for a specific PRD value, the tolerated error is greater in segments with higher average amplitudes as long as larger amplitudes contribute to higher values of the denominator of (1). As a result, the PRD hardly provides information about the absolute signal deviation between the original signal and the reconstructed one. This is illustrated with the example of figure 1 where two distinct signals are subjected to compression‡ for an identical reconstruction quality target set to be $\mathrm{PRD}_{target} = 2\%$. When analysing the Root Mean Square (RMS) value in one segment ($N = 1024$):

$$x_{rms,N}(n) = \frac{\|\mathbf{x}_N(n)\|}{\sqrt{N}}, \tag{5}$$

which is proportional to the denominator of (1), the signal of figure 1(a) has three–fold less RMS than that of figure 1(b), so although the PRD values obtained in both cases are almost the same, the compression error of the signal with bigger RMS is significantly higher (see lower plot of both graphs). Thus, the interpretability of the PRD is troublesome, so it is difficult to rely on a metric that works that way.

Using alternative metrics to assess quality may somehow alleviate this problem.

‡ The compressor employed for this example is described later in section 3.

Thus, we can use the normalized version of the PRD (PRDN):

$$\mathcal{D}_{0,N}(n) = \frac{\|\mathbf{e}_N(n)\|}{\|\mathbf{x}_N(n) - \bar{x}_N(n)\mathbf{1}_N\|} \cdot 100, \tag{6}$$

where $\bar{x}_N(n)$ denotes the mean value of the vector $\mathbf{x}_N(n)$ at time lag n, and $\mathbf{1}_N$ is an N–size column vector of ones. Basically, the PRDN consists of determining the PRD over a zero mean ECG vector, which minimizes the problem described above, though the dependence on the magnitude of the analysed signal segment still persists. However, the major drawback of this metric is that it may not be used to guarantee the quality target for the signal, as we will see later in section 4.

One way to eliminate the dependence on the signal magnitude would be to focus on the numerator of (1) or (6), which in fact is a scaled value of the RMSE:

$$e_{rms,N}(n) = \frac{\|\mathbf{e}_N(n)\|}{\sqrt{N}} = \frac{\mathcal{D}_N(n) \cdot x_{rms,N}(n)}{100}. \tag{7}$$

One qualitative advantage of the RMSE with regard to the previous metrics is that it assesses the coding error magnitude in absolute units (volts), providing a more comprehensible figure for waveform comparison.

In addition to global measures of quality, local analysis is of highest importance to find out, for instance, whether specific electrical patterns, such as the cardiac activation of a heartbeat, are within normal range. Thus, it is of big interest to confine significant local deviations to the minimal value as possible. In order to assess local effects, the maximum difference in voltage units is also used through the maximum amplitude error (MAX) (Blanco–Velasco et al. 2005), which is defined as:

$$\mathcal{E}_N(n) = \max_{\forall n} \{|\mathbf{e}_N(n)|\}. \tag{8}$$

Finally, the measure of the compression is reported by the CR, which indicates the bit reduction. Its analytical expression is

$$\mathcal{R}_N(n) = \frac{b_{x,N}(n)}{b_{c,N}(n)}, \tag{9}$$

where $b_{x,N}(n)$ and $b_{c,N}(n)$ are the amount of bits needed for the original and the compressed signal representation, respectively, in an N–samples long segment.

3. Compression Schemes

Lossy compression based on thresholding has received much attention because it achieves excellent results in terms of compression ratio, efficient implementation and quality of the reconstructed signal (Blanco–Velasco, Cruz–Roldán, López, Ángel M. Bravo & Martínez 2004, Blanco–Velasco, Cruz–Roldán, Godino–Llorente & Barner 2004, Chen et al. 2006, Benzid et al. 2007, Blanco–Velasco et al. 2007, Ouamri & Nat-Ali 2007, Benzid et al. 2008, Aggarwal & Patterh 2012, Abo–Zahhad et al. 2013). In this work, a thresholding–based compression scheme that relies on the retained energy for quality control is proposed. The compressor, which is shown in figure 2, consists of four steps:

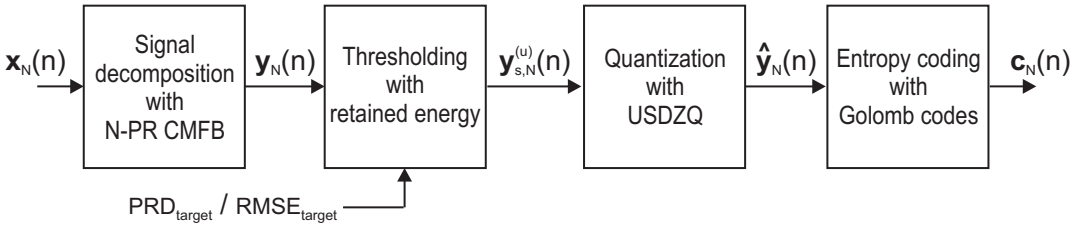


Figure 2. Block diagram of the proposed compression scheme.

signal decomposition, thresholding (which includes the control of quality), quantization and entropy encoding. The RMSE is proposed as the target metric to guarantee the quality of the reconstructed signal in the defined compression scheme. However, the other direct metrics under study, namely the PRD–based measures, can be also employed for this same function. In this way, the compressor is utilized later in section 4 to carry out a practical study on the use of direct metrics to control the reconstruction quality.

3.1. Signal decomposition

The energy of the ECG is not uniformly distributed in the frequency domain, so the signal is firstly decomposed into subband signals that can be quantized with different precision. This is accomplished here by means of Nearly–Perfect Reconstruction Cosine Modulated Filter Banks (N–PR CMFB). N–PR CMFB are a subclass of modulated M–channel maximally decimated filter banks that can be derived from a unified scheme to obtain different modulated filter banks as in (Cruz–Roldán, Martín, Sáez–Landete, Blanco–Velasco & Saramaki 2009). In this work, the number of channels ($M = 16$) and the L=192–length prototype filter are the same as in (Blanco–Velasco, Cruz–Roldán, López, Ángel M. Bravo & Martínez 2004), because they yield good performance when compared with similar schemes based on the Discrete Wavelet Transform (DWT) (Daubechies 1988).

3.2. Thresholding

The second stage is aimed to find a threshold value to discard non–significant samples from the point of view of the quality of the reconstructed signal. For this purpose, we adapt a method based on the retained energy that is implemented with the PRD in (Bazán-Prieto, Blanco-Velasco, Cárdenas-Barrera & Cruz-Roldán 2012b). This technique determines the threshold value under the condition that the energy captured by the significant samples, so called the retained energy, is sufficient to guarantee an *a priori* chosen quality target. In this work, we propose the RMSE as the parameter to control the quality in an application of cardiac signal compression.

In compressing an N–samples long vector $\mathbf{x}_N(n)$, we define the RMSE in terms of energy

$$e^2_{rms,N}(n) = \frac{\|\mathbf{e}_N(n)\|^2}{N} = \frac{E_{L,N}(n)}{N}, \tag{10}$$

where $E_{L,N}(n)$ is the energy of the tolerated error, which corresponds to the energy lost during the thresholding process. The retained energy after compression, $E_{R,N}(n)$, i.e., the energy of the compressed signal, is expressed as the difference between the total energy $E_{T,N}(n)$ and the lost energy $E_{L,N}(n)$

$$E_{R,N}(n) = E_{T,N}(n) - E_{L,N}(n), \tag{11}$$

where $E_{T,N}(n) = \|\mathbf{x}_N(n)\|^2 = \sum_{l=0}^{N-1} x^2(n-l)$ is known. Combining (10) and (11), we obtain

$$E_{R,N}(n) = E_{T,N}(n) - N \cdot e^2_{rms,N}(n). \tag{12}$$

This expression determines the energy that is retained for a specific distortion value given in terms of the RMSE. With this relation, the main issue consists of determining the set of significant samples that contributes to the retained energy in the subband domain. Let $\mathbf{y}_N(n) = [y(n), y(n-1), \cdots, y(n-N+1)]^T$ be the N dimensional vector obtained by concatenating the M subband signals. We obtain the set of sorted subband samples

$$\mathbf{y}_{s,N}(n) = [y_{(1)}, y_{(2)}, \cdots, y_{(N)}]^T, \tag{13}$$

where $|y_{(1)}| \geqslant |y_{(2)}| \geqslant \cdots \geqslant |y_{(N)}|$. If the quality target is chosen as an *a priory* RMSE value, the corresponding energy to be retained for guaranteeing that quality value is obtained by means of (12) ($e_{rms,N}(n) = \mathrm{RMSE}_{target}$). The energy can then be described as follows:

$$E_{T,N}(n) = E_{R,N}(n) + E_{L,N}(n) = \sum_{i=1}^{u} y^2_{(i)} + \sum_{i=u+1}^{N} y^2_{(i)}, \tag{14}$$

where the integer $u \in [1, N]$ corresponds to the first u most significant samples of the sorted vector $\mathbf{y}_{s,N}(n)$ that contributes to the retained energy. The threshold value is chosen to be $\varepsilon = \left|y_{(u)}\right|$. Thus, the $u \times 1$ vector containing the significant samples that are subsequently quantized and entropy encoded in the next stages is obtained:

$$\mathbf{y}^{(u)}_{s,N}(n) = [y_{(1)}, y_{(2)}, \cdots, y_{(u)}]^T. \tag{15}$$

3.3. Quantization and entropy coding

For the last two stages, a method that has reported superior performance in thresholding–based ECG compression is chosen (Chen et al. 2006). It firstly applies a uniform scalar dead zone quantization (USDZQ) to the significant coefficients stored in $\mathbf{y}^{(u)}_{s,N}(n)$ to obtain the equivalent sequence of quantized values $\hat{\mathbf{y}}^{(u)}_{s,N}(n)$. In this way, a vector $\hat{\mathbf{y}}_N(n) = [\hat{y}(n), \hat{y}(n-1), \cdots, \hat{y}(n-N+1)]^T$ of size $N \times 1$ with all the quantized subband samples is then recovered to perform the entropy coding. It comprises both the thresholded values and the quantized nonzero coefficients in the same positions as in $\mathbf{y}_N(n)$. Finally, Golomb codes are used to encode the samples of the sequence $\hat{\mathbf{y}}_N(n)$ into a set of codewords $\mathbf{c}_N(n)$ in the same way as it is described in (Chen et al. 2006).

4. Quality and compression study

4.1. Materials and methods

It has been shown in previous works that the use of a reduced group of signals overestimates the performance of the compression algorithms (Hernando–Ramiro, Blanco–Velasco, Cruz–Roldán & Pedroviejo–Benito 2011), so the experiments with the compressors are carried out with the first lead of all the records available in the MIT–BIH Arrhythmia Database (Moody & Mark 2001). These signals were acquired with a sampling frequency of 360 Hz and a resolution of 11 bits per sample over a 10 mV range. In addition, a 1024–baseline was added to each lead for storage purposes, so it is removed before processing.

The purpose of the compressors is to attain the predefined quality, so the encoding process finishes when the quality target is accomplished. In order to achieve this specification, we have chosen a quality tolerance of 2% with respect to the target value for the whole set of experiments conducted in this work.

The signal is processed in finite consecutive non–overlapping segments whose size has an impact in several technological aspects, namely, real–time implementation, size and cost of the devices, and the response delay of the system. A block length of $N = 1024$ samples is a reasonable solution which has already been proposed in several works (Benzid et al. 2007, Blanco–Velasco et al. 2007, Ouamri & Nat-Ali 2007, Hernando–Ramiro et al. 2011). Zero–padding the last segment contributes to an artificially increased CR, so this effect is avoided by removing this segment.

In evaluating the performance of a compression system, the theoretical study of the Shannon entropy can report the maximum CR value (Hernando–Ramiro, Blanco–Velasco, Moreno–Martínez, Cruz–Roldán & Sáez–Landete 2009), although it does not reveal the true capability of a system. In this work, encoders and decoders are separated so as to generate the actual bitstream, reporting thus results that are realistic and close to the technological implementation.

4.2. Analysis of direct metrics

The direct metrics described in section 2 are initially analysed with record 117 from the MIT–BIH Arrhythmia Database using the PRD, the PRDN, and the RMSE as target parameters, so three trials are developed, one per metric. It must be noted that the compressor used to evaluate the PRDN follows the same scheme as presented in section 3 but substituting the type of quality target. The signal is split in blocks of $N = 1024$ samples for a total of $K = 634$ segments, and the results are shown in table 1. The scores for the entire signal (with subscripts KN), as well as the mean values across segments, denoted by the mathematical expectation $\mathrm{E}\{\cdot\}$, are given. The standard deviations are also included. In order to get fair comparison, the same quality measurement must be achieved in the three compression trials. Therefore, we performed as follows: 1) in the first trial, the quality target is set as $\mathrm{PRD}_{target} = 2\%$, obtaining the results shown in

the second column of table 1; 2) the results of the entire signal are used to define the targets for the remaining metrics.

The results shown in the third column correspond to the PRDN taken as target. We may see that for an *a priori* quality defined to be 7.48%, the average PRDN per segment stands close to it (7.57%), but that of the entire signal provides a much different value (6.66%). The obtained quality for the whole signal in terms of the PRDN present a deviation of 11% with respect to the target value. Regarding the two other metrics, namely the PRD and the RMSE, the quality of the whole signal as well as the average values per segment provide similar results, in fact, the deviation from the target value is about 1% in both cases. Therefore, the PRDN does not comply by far with the maximum tolerance allowable of 2% while the other two metrics do.

Table 1. Quality results for the signal 117 of the MIT–BIH Arrhythmia Database.

	$\text{PRD}_{target} = 2\%$	$\text{PRDN}_{target} = 7.48\%$	$\text{RMSE}_{target} = 17.46\mu\text{V}$
$\mathcal{D}_{KN}(KN-1)$	**2.02**	1.80	2.04
$\mathcal{D}_{0,KN}(KN-1)$	7.48	**6.66**	7.55
$e_{rms,KN}(KN-1)$	17.46	15.55	**17.62**
$\text{E}\{\mathcal{D}_N(n)\} \pm \sigma$	$\mathbf{2.02} \pm 0.05$	1.79 ± 0.42	2.09 ± 0.32
$\text{E}\{\mathcal{D}_{0,N}(n)\} \pm \sigma$	8.80 ± 1.37	$\mathbf{7.57} \pm 0.15$	8.98 ± 1.04
$\text{E}\{e_{rms,N}(n)\} \pm \sigma$	$17.32 \pm \mathbf{2.22}$	$15.15 \pm \mathbf{3.50}$	$\mathbf{17.61 \pm 0.39}$

The experiment above is carried out for the 48 signals of the MIT–BIH Arrhythmia Database in order to generalize the metrics behaviour. The results are shown in figure 3 in terms of quality deviation. In the upper panel, which depicts the quality deviation of the entire signal with respect to the quality target, the PRDN exhibits lack of stability and consistency as the deviation varies significantly from one signal to another. In addition, in several cases the PRDN deviation levels exceed the specified tolerance of 2%, while the other two parameters, the PRD and the RMSE, provide regular deviations around the required 2%. Therefore, the PRDN cannot be used to ensure the quality of the ECG signal after compression.

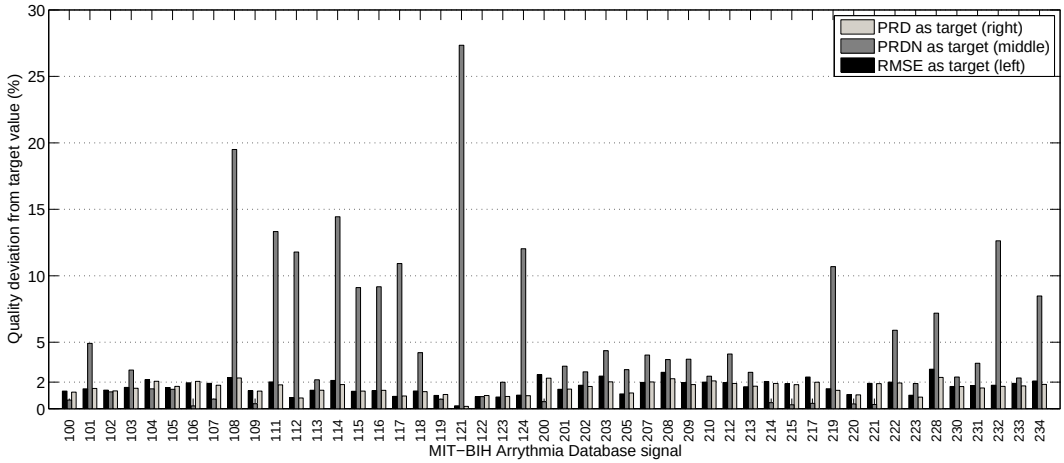
The reason for this behavior is because the mean value of each processing block is different. Let $\bar{x}_N(n + iN)$ be the mean value of the $(i-1)$–th segment. As $\bar{x}_N(n + iN) \neq \bar{x}_N(n + jN)$, $\forall i \neq j$, this is equivalent, at the effects, to process a signal with discontinuities at the border of each segment due to the mean removal. Regarding the target PRDN, it can be attained for all the segments:

$$\mathcal{D}_{0,N}(n + iN) = \text{PRDN}_{target},\ i = 0, 1, \cdots, K-1, \tag{16}$$

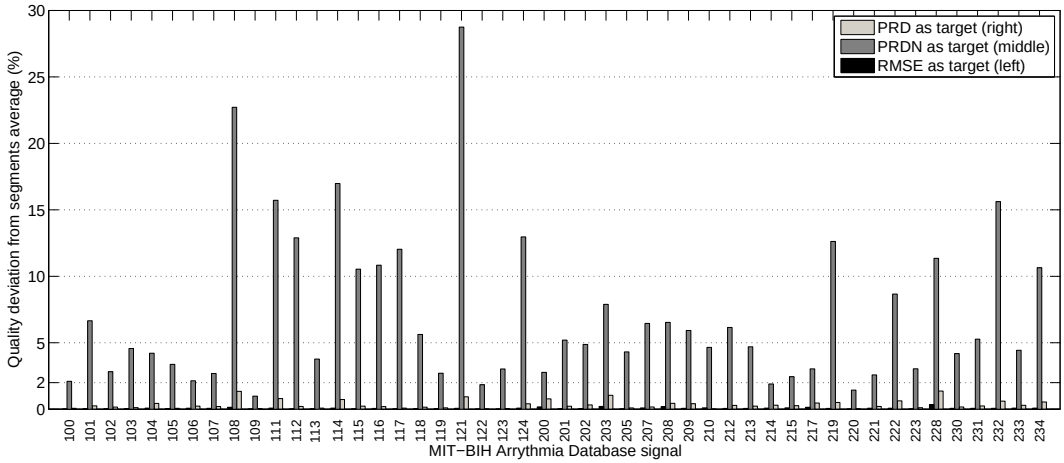
because they are processed separately. Nevertheless, in regard to the full PRDN, as it is evaluated over the entire signal, $\mathbf{x}_{KN}(n + (K-1)N)$, which does not present those discontinuities, it results in a value that does not necessarily match the objective:

$$\mathcal{D}_{0,KN}(n + (K-1)N) \neq \text{PRDN}_{target}. \tag{17}$$

This issue is clearly observed in figure 3(b), which depicts the quality deviation of the entire signal with respect to the mean quality across segments. We can see huge



(a) Deviation of quality with respect to the required target values.



(b) Deviation of quality with respect to the mean quality value across segments.

Figure 3. Study of the quality consistency of the different metrics for every signal of the MIT–BIH Arrhythmia Database (signal identifiers in the x–axis).

deviations when the PRDN is used as metric, and small ones for the other two figures of merit. In fact, using the PRD and the RMSE, the deviations are less than 1.5% in all of the cases, complying on average with the specified maximum tolerance of 2%. Therefore, the use of the PRD or the RMSE assures that the quality control of the segments is consistent with the overall one.

On the other hand, as the RMSE informs about the magnitude deviation of the reconstructed signal with respect to the original one, its standard deviation is likely to provide a good indication of the variability of the error. Thus, we see in the last row of table 1 that the lowest value is obtained when the RMSE is used to control the quality. This trend is extended to all of the signals of the MIT–BIH Arrhythmia Database, as it can be observed in figure 4.

Finally, to find out which metric, either the PRD or the RMSE, produces less local error, we analyse the variability of the MAX error, so for this purpose we focus on

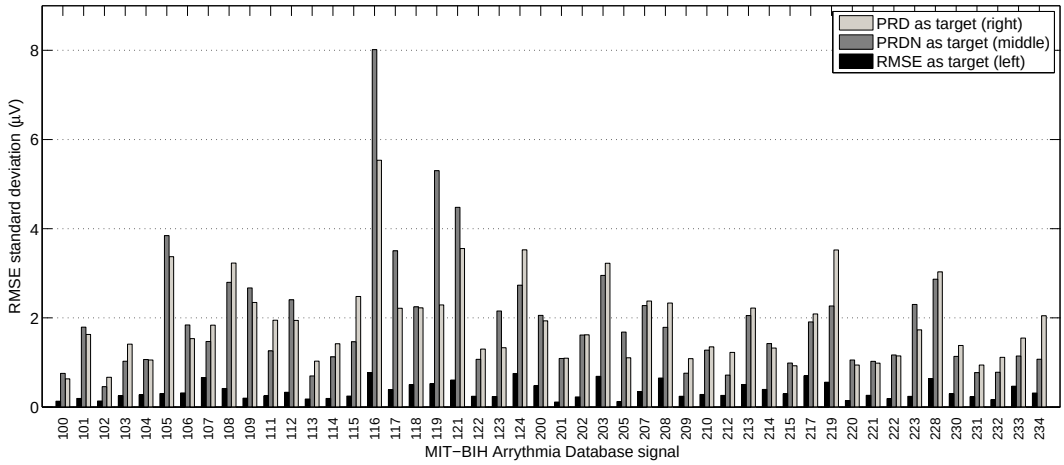


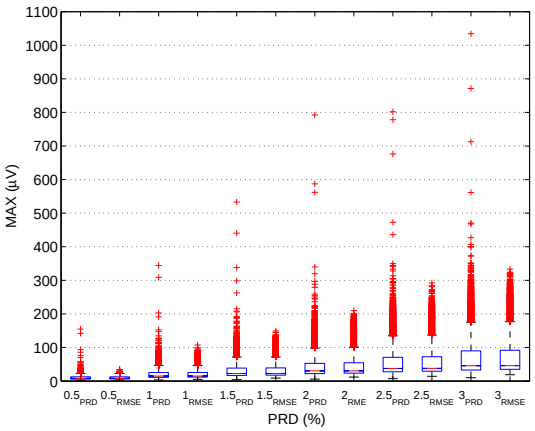
Figure 4. RMSE standard deviation $\sigma\{e_{rms,N}(n)\}$ for all of the segments of every signal of the MIT–BIH Arrhythmia Database (signal identifiers in the x–axis).

the density functions. The variable $\mathcal{E}_N(n)$ (8) is determined for all the signals of the MIT–BIH Arrhythmia Database again in two trials: 1) using the PRD as target, and 2) using the RMSE as target. For comparison purposes, in both cases, the *a priori* targets are adjusted to provide the same quality measurement given in terms of PRD. Figure 5(a) shows the corresponding box plot diagrams and they all exhibit right sided tails. In order to evaluate which tails are heavier, we use the skewness, which is reported in figure 5(b). Right tails correspond to positive values of skewness and the results reveal higher magnitudes when the PRD is taken as target, which means that the tails of the MAX error distributions are heavier when the PRD is used as target. This analysis manifests that the PRD is less adequate to constrain the local errors, so if we desire minimizing local errors on ECG signals retrieved after compression, the use of the RMSE gives rise to a more suitable technique.

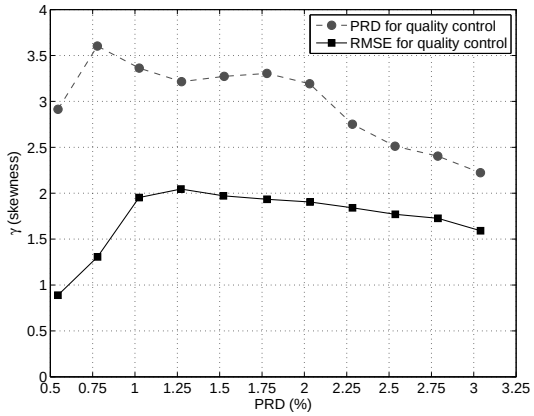
4.3. Analysis of compression

This section analyses the compression performance provided by the different quality control metrics studied in section 4.2. Our aim is to preserve good quality for the reconstructed signal with the highest CR as possible, but increasing the CR at the expense of quality is not an option. Thus, the simulations are constrained to high quality cases.

Given that the PRD has been more extensively used, we first analyse the rate–distortion curves using this figure of merit. In order to fulfil our purposes of high quality reconstruction, we establish low PRD target values in our experiments: from 0.5% to 3% in steps of 0.25%. The compression results as a function of the PRD outcome are depicted in figure 6. It may be seen that the compressor that employs the RMSE for quality control outperforms the PRD–based one for the same levels of quality requirements. Thus, the introduction of the RMSE as target metric, not only does not



(a) Distribution of $\mathcal{E}_N(n)$. The subscripts at the horizontal axis stands for the metric used as target.



(b) Skewness of $\mathcal{E}_N(n)$.

Figure 5. Variability study of the MAX value per segment ($N = 1024$) for the whole MIT–BIH Arrhythmia Database.

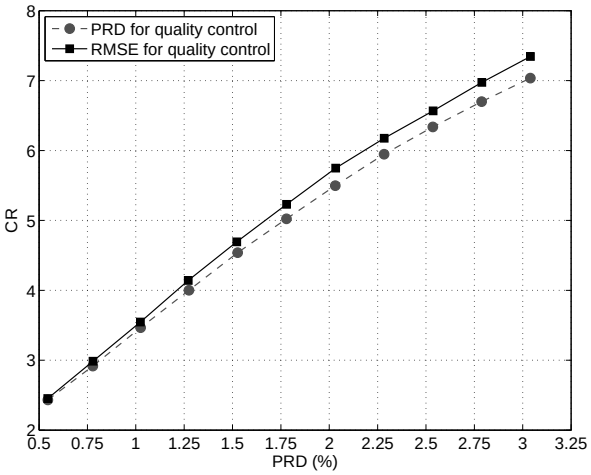
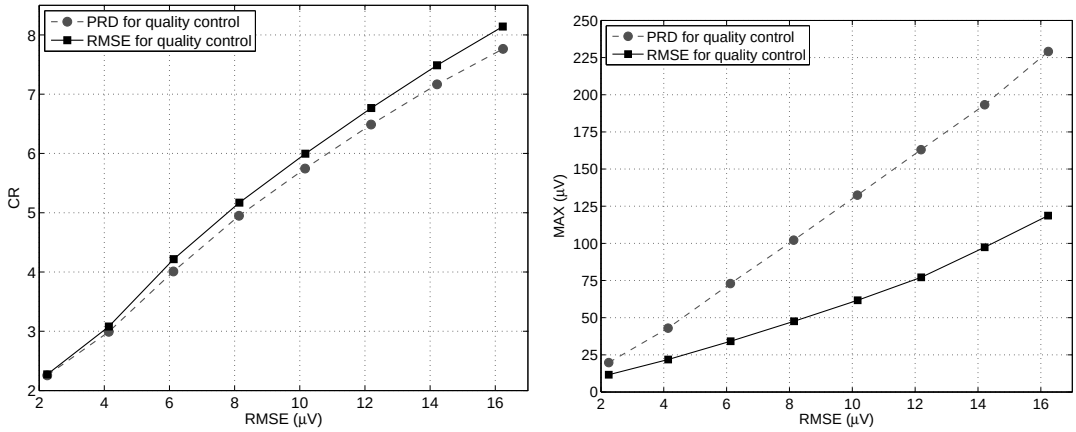


Figure 6. Performance of the proposed compressors: $\mathrm{E}\{\mathcal{R}_N(n)\}$ as a function of $\mathrm{E}\{\mathcal{D}_N(n)\}$.

imply any reduction of the CR, but it even tends to give rise to higher compression.

On the other hand, the rate–distortion curves in terms of the RMSE are shown in figure 7(a). The range of RMSE target values are set between 2 μV and 16 μV in steps of 2 μV. These quality levels stand for very low distortions, even close to the technical resolution limit, for a signal whose amplitude ranges around units of millivolts. The same conclusions as drawn from the PRD results can be derived from these new curves. In addition, the local distortion is analysed in figure 7(b) and we can clearly appreciate that the use of the RMSE to control the quality considerably reduces the local errors, in concurrence with the analysis of section 4.2. Finally, table 2 shows the exact values of CR and MAX along with the standard deviation when the RMSE is used as target.



(a) $\mathrm{E}\{\mathcal{R}_N(n)\}$ as a function of $\mathrm{E}\{e_{rms,N}(n)\}$. (b) $\mathrm{E}\{\mathcal{E}_N(n)\}$ as a function of $\mathrm{E}\{e_{rms,N}(n)\}$.

Figure 7. Performance of the proposed compressors.

Table 2. Compression performance corresponding to the results of Figure 7 with standard deviation.

Quality control	$\mathrm{RMSE}_{target}(\mu V)$	2	4	6	8	10	12	14	16
PRD	$\mathrm{E}\{\mathcal{R}_N(n)\}$	2.26	2.99	4.01	4.95	5.75	6.49	7.17	7.76
	$\sigma_{\mathcal{R}}$	0.20	0.34	0.58	0.76	0.92	1.07	1.20	1.27
	$\mathrm{E}\{\mathcal{E}_N(n)\}(\mu V)$	19.8	43.0	73.0	102	132	163	193	229
	$\sigma_{\mathcal{E}}(\mu V)$	7.80	22.9	47.0	64.6	78.3	88.3	94.8	115
RMSE	$\mathrm{E}\{\mathcal{R}_N(n)\}$	2.27	3.08	4.22	5.17	6.00	6.77	7.49	8.14
	$\sigma_{\mathcal{R}}$	0.20	0.37	0.63	0.81	0.97	1.13	1.24	1.37
	$\mathrm{E}\{\mathcal{E}_N(n)\}(\mu V)$	11.6	21.8	34.1	47.6	61.7	77.1	97.4	119
	$\sigma_{\mathcal{E}}(\mu V)$	1.43	2.17	4.15	8.54	13.3	14.7	20.8	27.8

5. Conclusion

Direct metrics are extensively employed in the design of ECG compression methods for a wide range of practical applications. The main advantage of these figures of merit is that they can be directly used on the ECG signal. In this way, no additional processing, which is usually a source of error, needs to be applied to the recorded medical data. Among the different direct metrics, only those that provide a consistent and reliable assessment must be used to assure a tight control of the reconstruction quality. In this sense, we have studied both the widely accepted PRD–based measures and the RMSE as direct metrics to evaluate the quality of ECG compressed signals. From a theoretical analysis, we propose the use of the RMSE, because it offers a quantitative idea about the absolute error introduced in each segment of the reconstructed signal while the PRD does not. The PRD reports, on average, a relative measure of the deviation with respect to the magnitude of the signal in that segment, so for a specific PRD the absolute deviation across segments may differ. Therefore, it is far easier to relate an RMSE value to a waveform feature, which may ease the comprehension of clinicians toward the use of compressed ECG signals.

Based on these facts, a novel scheme for thresholding–based ECG compression that guarantees the quality of the reconstructed signal by means of the retained energy is defined. The proposed compressor is employed to test the practical performance of the analysed metrics in terms of both quality control and compression. The results confirm that the use of the RMSE to assess quality permits to constrain the overall error along the whole ECG in a more meaningful and consistent way. In addition, the RMSE provides a tightest control of local errors, limiting both the magnitude and variability of the maximum error to lower values. It is revealed that these improvements are achieved without degrading the compression capabilities of the system. In fact, the compression ratio is even increased for the same quality levels when the RMSE is employed to control the quality instead of the PRD. Therefore, in view of the results arisen from this study, we may conclude that the RMSE is better suited as quality measure in ECG compression when it is accomplished by thresholding–based methods such as the one presented in this work, so we propose its use in this type of systems.

References

Abo–Zahhad, M., Al–Ajlouni, A. F., Ahmed, S. M. & Schilling, R. (2013). A new algorithm for the compression of ECG signals based on mother wavelet parameterization and best-threshold levels selection, *Digital Signal Processing* **23**(3): 1002–1011.

Aggarwal, V. & Patterh, M. S. (2012). Quality controlled ECG compression using essentially non–oscillatory point–value decomposition (ENOPV) technique, *Digital Signal Processing* **22**(6): 878–884.

Alesanco, A., García, J., Serrano, P., Ramos, L. & Istepanian, R. S. H. (2006). On the guarantee of reconstruction quality in ECG wavelet codecs, *Proc. of 28th IEEE EMBS Annual International Conference*, New York (USA), pp. 6461–6464.

Alesanco, A., Olmos, S., Istepanian, R. S. H. & García, J. (2006). Enhanced real-time ECG coder for packetized telecardiology applications, *IEEE Transactions on Information Technology in Biomedicine* **10**(2): 229–236.

Bazán-Prieto, C., Blanco-Velasco, M., Cárdenas-Barrera, J. & Cruz-Roldán, F. (2012a). Analysis of tractable distortion metrics for EEG compression applications, *Physiological Measurement* **33**(7): 1237–1247.

Bazán-Prieto, C., Blanco-Velasco, M., Cárdenas-Barrera, J. & Cruz-Roldán, F. (2012b). Retained energy-based coding for EEG signals, *Medical Engineering & Physics* **34**(7): 892–899.

Benzid, R., Marir, F. & Bougechal, N. E. (2007). Electrocardiogram compression method based on the adaptative wavelet coefficients quantization combined to a modified two-roled encoder, *IEEE Signal Processing Letters* **14**(6): 373–376.

Benzid, R., Messaoudi, A. & Boussaad, A. (2008). Constrained ECG compression algorithm using the block-based discrete cosine transform, *Digital Signal Processing* **18**(1): 56–64.

Blanco–Velasco, M., Cruz–Roldán, F., Godino–Llorente, J. I. & Barner, K. E. (2004). ECG compression with retrieved quality guaranteed, *Electronics Letters* **40**(23): 1466–1467.

Blanco–Velasco, M., Cruz–Roldán, F., Godino–Llorente, J. I. & Barner, K. E. (2007). Wavelet packets feasibility study for the design of an ECG compressor, *IEEE Transactions on Biomedical Engineering* **54**(4): 766–769.

Blanco–Velasco, M., Cruz–Roldán, F., Godino–Llorente, J. I., Blanco–Velasco, J., Armiens–Aparicio, C. & López, F. (2005). On the use of PRD and CR parameters for ECG compression, *Medical Engineering & Physics* **29**(9): 798–802.

Blanco–Velasco, M., Cruz–Roldán, F., López, F., Ángel M. Bravo & Martínez, D. (2004). A

low computational complexity algorithm for ECG signal compression, *Medical Engineering & Physics* **26**(7): 553–568.

Blanco–Velasco, M., Cruz–Roldán, F., Moreno–Martínez, E., Godino–Llorente, J. I. & Barner, K. E. (2008). Embedded filter bank-based algorithm for ECG compression, *Signal Processing* **88**(4): 1402–1412.

Capua, C. D., Meduri, A. & Morello, R. (2010). A smart ECG measurement system based on web-service-oriented architecture for telemedicine applications, *IEEE Transactions on Instrumentation and Measurement* **59**(10): 2530–2538.

Chen, J. & Itoh, S. (1998). A wavelet transform–based ECG compression method guaranteeing desired signal quality, *IEEE Transactions on Biomedical Engineering* **45**(12): 1414–1419.

Chen, J., Ma, J., Zhang, Y. & Shi, X. (2006). ECG compression based on wavelet transform and Golomb coding, *Electronics Letters* **42**(6): 322–324.

Cruz–Roldán, F., Martín, P., Sáez–Landete, J., Blanco–Velasco, M. & Saramaki, T. (2009). A fast windowing-based technique exploiting spline functions for designing modulated filter banks, *IEEE Transactions on Circuits and Systems–I: Regular Papers* **56**(1): 168–178.

Daubechies, I. (1988). Orthonormal bases of compactly supported wavelets, *Communications on pure and applied Mathematics* **41**(7): 909–996.

dos Santos Guimaraes, F., Lovisolo, L., Blanco–Velasco, M. & Cruz–Roldán, F. (2010). On the compression of ECG records employing triangular elements and analysis–by–synthesis modeling, *Proc. of 2010 IEEE International Symposium on Circuits and Systems (ISCAS)*, Vol. 1, pp. 3084–3087.

Engin, M., Caglav, E. & Engin, E. (2005). Real-time ECG signal transmission via telephone network, *Measurement* **37**(2): 167–171.

Hernando–Ramiro, C., Blanco–Velasco, M., Cruz–Roldán, F. & Pedroviejo–Benito, F. (2011). Efficient thresholding-based ECG compressors for high quality applications using cosine modulated filter banks, *Proc. of the 33rd Annual International Conference of the IEEE Engineering in Medicine and Biology Society (EMBC 2011)*, Boston (USA), pp. 7079–7082.

Hernando–Ramiro, C., Blanco–Velasco, M., Moreno–Martínez, E., Cruz–Roldán, F. & Sáez–Landete, J. (2009). Efficient source coding in a thresholding-based compressor using the discrete wavelet transform, *Proc. of International Conference on Bio-Inspired Systems and Signal Processing (BIOSIGNALS 2009)*, Porto (Portugal), pp. 259–264.

Hung, K.-C., Tsai, C.-F., Ku, C.-T. & Wang, H.-S. (2009). A linear quality control design for high efficient wavelet-based ECG data compression, *Computer Methods and Programs in Biomedicine* **94**(2): 109–117.

Iliev, I., Krasteva, V. & Tabakov, S. (2007). Real-time detection of pathological cardiac events in the electrocardiogram, *Physiological Measurement* **28**(3): 259–276.

Ku, C.-T., Hung, K.-C., Wu, T.-C. & Wang, H.-S. (2010). Wavelet-based ECG data compression system with linear quality control scheme, *IEEE Transactions on Biomedical Engineering* **57**(6): 1399–1409.

Moody, G. B. & Mark, R. G. (2001). The impact of the MIT–BIH Arrhythmia Database, *IEEE Engineering in Medicine and Biology* **20**(3): 45–50.

Ouamri, A. & Nat-Ali, A. (2007). ECG compression method using lorentzian functions model, *Digital Signal Processing* **17**(1): 319–326.

Sayood, K. (2000). *Introduction to Data Compression*, 2nd edn, Morgan Kaufmann Publishers, San Francisco (Estados Unidos).

Zigel, Y., Cohen, A. & Katz, A. (2000a). ECG signal compression using analysis by synthesis coding, *IEEE Transactions on Biomedical Engineering* **47**(10): 1308–1313.

Zigel, Y., Cohen, A. & Katz, A. (2000b). The Weighted Diagnostic Distortion (WDD) measure for ECG signal compression, *IEEE Transactions on Biomedical Engineering* **47**(11): 1422–1430.