PROBLEMS OF CHAPTER 1: Transient Circuits.

February 8, 2018

Problem 1.1

For the circuit in Figure 1 the switch opens at $t = 0$. We know that the switch was closed from $t = -\infty$. Determine the evolution in time of the voltage of the inductor ($v_L(t)$) as a function of the components (E, L, R_1 and R_2).

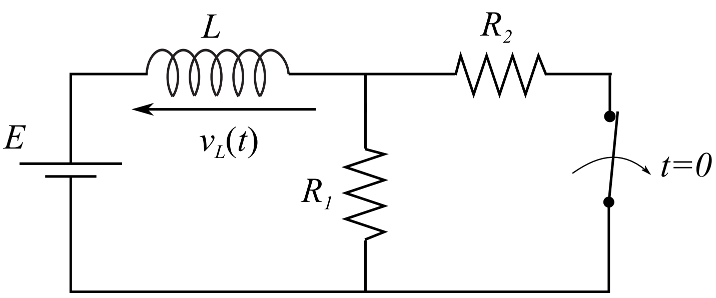


Figure 1

Result

a) $v(t) = \begin{cases} 0 & ; \quad t < 0 \\ -\dfrac{E \cdot R_1}{R_2} \cdot e^{-\dfrac{R_1}{L}t} & ; \quad t > 0 \end{cases}$

Problem 1.2

The circuit in Figure 1 can be used to measure the speed of a bullet. The bullet interrupts the current on the battery, when it breaks the source P_1. At that moment, the capacitor starts to discharge with R until the bullet breaks the surface P_2, at t_0. At that instant, the discharge process of the capacitor ends. When the bullet breaks P_2 the voltmeter, with an infinite internal resistor, measures 100 V.

Determine:

a) Time-domain expression of the voltage of the capacitor since the instant of time that the bullets breaks the surface P_1, until the bullets destroys the surface P_2.

b) Time-domain expression of the voltage of the capacitor from the instant of time in which the bullets breaks the surface P_2.

c) Time in which the bullet travels from P_1 to P_2.

d) Speed of the bullet if the distance between P_1 and P_2 is of 6 cm.

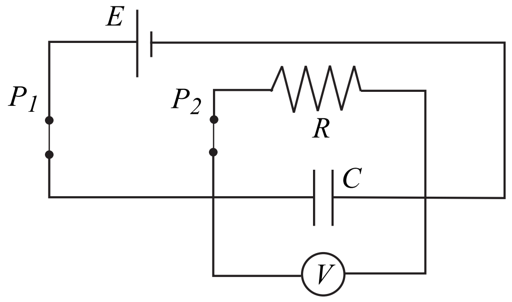


Figure 1

$$Data: \quad C = 1\ \mu F \quad ; \quad R = 1\ k\Omega \quad ; \quad E = 120\ V$$

Result

a) $v_C(t) = 120 \cdot e^{-10^3 t} \cdot (u(t) - u(t - t_0))\ V$

b) $v_C(t) = 100 \cdot u(t - t_0)\ V$

c) $t_0 = 182,3\ \mu s$

d) $v = 1184.7\ km/h$

Problem 1.3

In the circuit of the figure 1, the switcher is in position (1) since $t = -\infty$. At $t = 0$, the switcher changes to position (2) remaining in this position hence forth. Calculate:

a) Initial conditions of the elements that store energy at the switching instant.

b) Expression of the Laplace transform of the current $i(t)$ for $t > 0$.

c) Expression of $i(t)$ of $t > 0$.

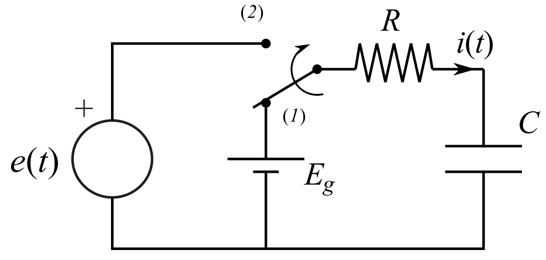


Figure 1

Data: $R = 1\ \Omega$; $C = 1$ F; $e(t) = \cos(t)$ V; $E_g = 1$ V

Result

a) $v_C(t=0^-) = E_g = 1$

b) $I(s) = -\dfrac{1}{(s^2+1)\,(s+1)}$

c) $i(t) = -\dfrac{1}{2} \cdot e^{-t} + \dfrac{1}{\sqrt{2}} \cdot \sin\left(t + \dfrac{3\pi}{4}\right)$ A ; $t > 0$

Problem 1.4

In the circuit of Figure 1, the switch S_1 is in position (1) and the switch S_2 is closed since $t = -\infty$. At $t = 0$ s, the switch S_1 changes to position (2) and the switch S_2 is opened.

a) At the switching instant $t = 0$, determine the initial conditions of the elements that store energy.

b) Compute the Laplace transform of the voltage $v(t)$ and the current $i_C(t)$, for $t > 0$ s.

c) Time-domain expression of the voltage $v(t)$ for $t > 0$ s.

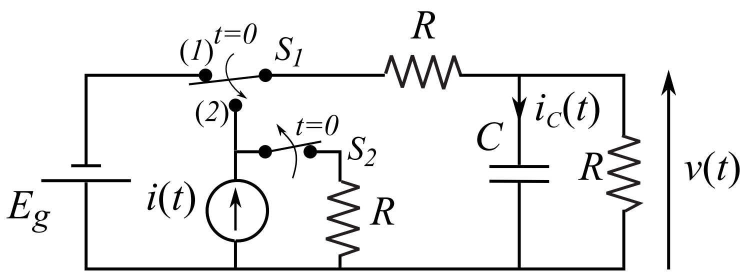


Figure 1

Data: $E_g = 2$ V ; $i(t) = \sin(t)$ A ; $R = 1\,\Omega$; $C = 1$ F

Result

a) $v_C(0^-) = -1$ V

b) $V(s) = -\dfrac{s^2}{(s^2+1)\cdot(s+1)}$; $I_C(s) = \dfrac{s^2+s+1}{(s^2+1)\cdot(s+1)}$

c) $v(t) = -\dfrac{e^{-t}}{2} - \dfrac{1}{\sqrt{2}} \cdot \sin\left(t + \dfrac{3\pi}{4}\right) = \dfrac{1}{\sqrt{2}} \cdot \sin\left(t - \dfrac{\pi}{4}\right) - \dfrac{e^{-t}}{2}$ V

Problem 1.5

For the circuit in Figure 1, the switch S_1 is open and the switch S_2 is in position (2) since $t = -\infty$. At $t = 0$ s switch S_1 is closed and switch S_2 commutes to position (1). At $t = \frac{1}{2}$ s switch S_2 commutes to position (2). With these conditions:

a) Determine the temporal evolution of voltage $v(t)$ from $t = 0\ s$ to $t = \frac{1}{2}\ s$.

b) Compute the temporal evolution of voltage $v(t)$ once the switch S_2 is open.

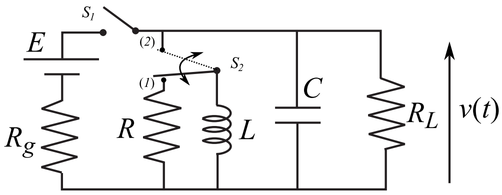


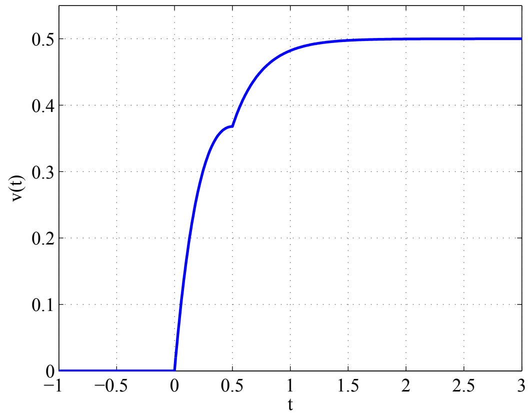
Figure 1

$$Data: \quad L = 1\ H \quad ; \quad C = \frac{1}{4}\ F \quad ; \quad R_g = R_L = 2\ \Omega \quad ; \quad E = 1\ V$$

Result

a) $v(t) = 2t \cdot e^{-2t} \cdot \left(u(t) - u\left(t - \frac{1}{2}\right)\right)$

b) $v(t) = \left[\frac{1}{2} - \left(\frac{1}{2} - \frac{1}{e}\right) \cdot e^{-4 \cdot \left(t - \frac{1}{2}\right)}\right] \cdot u\left(t - \frac{1}{2}\right)$



Problem 1.6

For the RLC circuit shown in Figure 1 (a) we know that it contains with one resistor, one inductor and one capacitor. We know as well that the switch is position (1) since $t = -\infty$, and that the switch conmmutes to position (2) at $t = 0$. The response of the circuit, for $v(t)$, is shown in Figure 1 (b). Obtain one possible internal structura for the circuit.

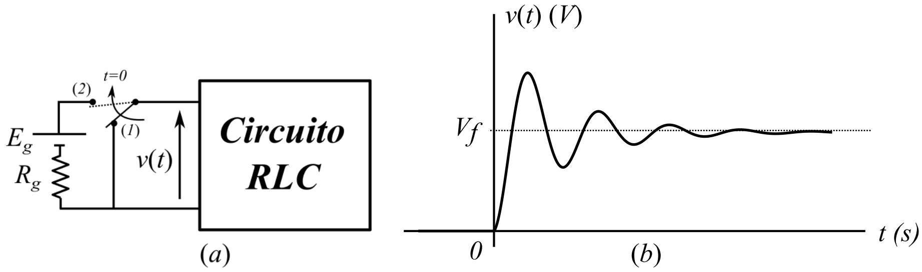
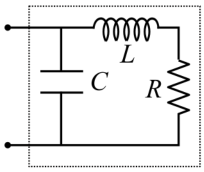


Figure 1

Result



Problem 1.7

For the circuit of the Figure 1 the switch is closed. If we open the switch at $t = 0\ s$, determine:

a) Current in the inductor $i_L(t)$ and the voltage of the capacitor, $v_C(t)$, just before we open the switch.

b) Laplace's transform expression of $v_C(t)$ for $t > 0\ s$.

c) Undamped natural frequency, ω_n, and damping coefficient, ξ, of the second order system. Justify the type of damping of the circuit.

d) Time-domain expression of the voltage in the capacitor, $v_C(t)$, for $t > 0\ s$.

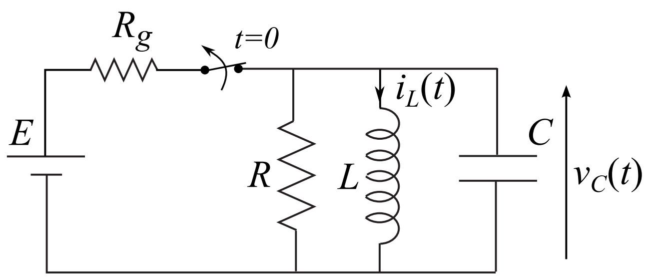


Figure 1

$$Data:\ \ E = 1\,V \ \ ; \ \ C = 1\,F \ \ ; \ \ L = 1\,H \ \ ; \ \ R_g = 1\,\Omega \ \ ; \ \ R = 1\,\Omega$$

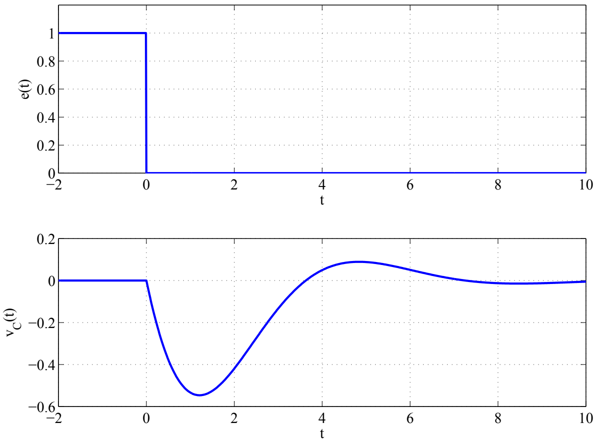
Result

a) $I_L(0^-) = 1\ A \quad ; \quad V_C(0^-) = 0\ V$

b) $V_C(s) = \dfrac{-1}{\left(s+\dfrac{1}{2}\right)^2 + \dfrac{3}{4}}$

c) $\omega_n = 1 \text{ rad/s} \quad ; \quad \xi = \frac{1}{2} \quad ;$ Underdamped system

d) $v_C(t) = \frac{2}{\sqrt{3}} \cdot e^{-\frac{t}{2}} \cdot sin\left(\frac{\sqrt{3}}{2}t + \pi\right) \cdot u(t)\ V$



Problem 1.8

For the circuit of the Figure 1, the switch is in the position (1). At $t_0 = \frac{\pi}{4}\ s$ the switch commutes to position (2):

a) Obtain the current of the inductor $i_L(t)$, for $t \geq t_0$

b) Determine the value of the resistor R, if L and C do not change, so as to have a critically damped current $i_L(t)$.

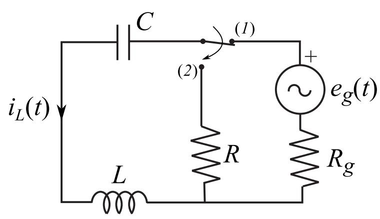


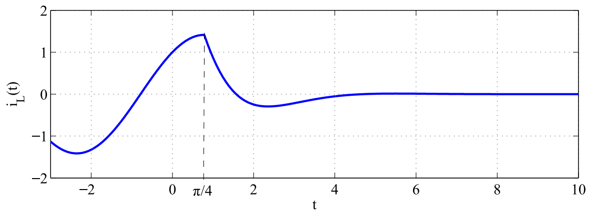
Figure 1

$$Data: \quad e_g(t) = 4 \cdot sin(t)\ V \quad ; \quad L = 2\ H \quad ; \quad C = \frac{1}{4}\ F \quad ; \quad R_g = 2\Omega \quad ; \quad R = 4\ \Omega$$

Result

a) $i_L(t) = 2 \cdot e^{-\left(t-\frac{\pi}{4}\right)} \cdot \sin\left(t + \frac{\pi}{2}\right) \cdot u\left(t - \frac{\pi}{4}\right)$

b) $R = 4\sqrt{2}\ \Omega$



Problem 1.9

For the circuit shown in Figure 1, the switch I_1 is in position since $t = -\infty\ s$. At $t = 0\ s$, this switch commutes to position 2. Switch I_2 is open until the instant of time $t = t_1\ s$, when the voltage of the capacitor is of 30 V. At this instant of time, t_1, switch I_2 is closed.

a) Determine this instant of time t_1 for which the voltage in the capacitor reaches the value of 30 V.

b) Finde the time-domain expression of the current $i_L(t)$ for $t > t_1$.

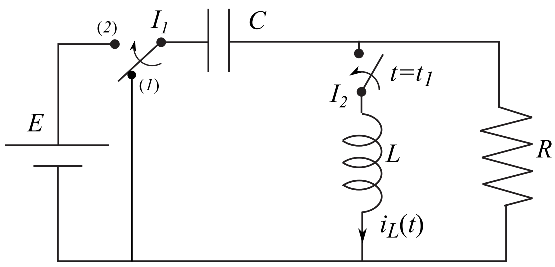


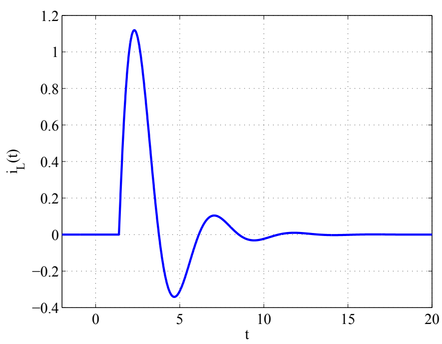
Figure 1

$$Data: \quad R = 8\ \Omega \quad ; \quad E = 40\ V \quad ; \quad L = 4\ H \quad ; \quad C = \frac{1}{8}\ F$$

Result

a) $t_1 = Ln(4)$

b) $i_L(t) = \frac{5}{\sqrt{7}} \cdot e^{-\frac{t-t_1'}{2}} \cdot \sin\left(\frac{\sqrt{7}}{2}(t - t_1)\right) \cdot u(t - t_1)\ A$



Problem 1.10

For the circuit of the Figure 1 the switch is open from $t = -\infty$ s. At $t = 0$, the switch is closed.Determine:

a) Value of the current in the inductor, $i_L(t)$, and the voltage of the capacitor, $v_C(t)$, for the instant $t = 0^-$. Value of the current of the capacitor and the voltage of the inductor for $t = 0^+$.

b) Proof that the Laplace's transform expression of the voltage of the capacitor is given by:

$$V_C(s) = \frac{\frac{5}{s} + 5(R_1 + s)}{s^2 + \frac{2R_1+1}{2}s + \frac{R_1+2}{2}}$$

c) Value of R_1 in order to have an undamped natural frequency of $\sqrt{2}$ rad/s , and the value fo the damping coefficient.

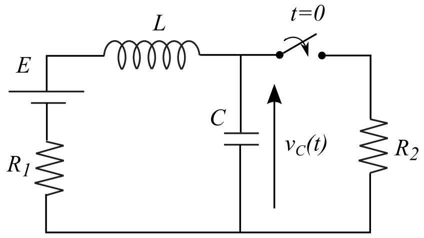


Figure 1

$$Data: \quad E = 5 \text{ V} \quad ; \quad L = 1 \text{ H} \quad ; \quad C = 1 \text{ F} \quad ; \quad R_2 = 2\,\Omega$$

Result

a) $\begin{cases} I_L(0^-) = 0 \\ V_C(0^-) = E = 5\ V \\ V_L(0^+) = 0 \\ I_C(0^+) = \frac{5}{2}\ A \end{cases}$

b) It is correct

c) $R_1 = 2\,\Omega$; $\xi = \frac{5\sqrt{2}}{8} < 1$ Under damped.

Problem 1.11

For the circuit of the Figure 1 the switch is closed from $t = -\infty$ s. At $t = 0$ s. the switch is open, remaining on this state indefinitely. Determine:

a) Initial conditions of the elements of the circuit able to save energy.

b) Expression of $I_L(s)$ as a function of R_2 for $t > 0$.

c) Value of R_2 in order to obtain a damping coefficient of $\xi = \dfrac{3}{4}$.

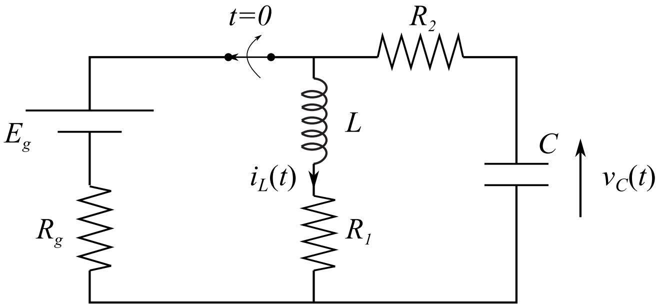


Figure 1

Data: $E_g = 5$ V; $R_g = 2\Omega$; $R_1 = 3\Omega$; $C = 0.1$ F; $L = 10$ H.

Result

a) $I_L(0^-) = 1\ A$, $V_C(0^-) = 3\ V$.

b) $I_L(s) = \dfrac{s + \dfrac{3}{10}}{s^2 + \dfrac{3 + R_2}{10}s + 1}$.

c) $R_2 = 12\ \Omega$.

Problem 1.12

For the circuit of the Figure 1, at $t = 0\ s$ the capacitor has a voltage of $v(t = 0) = 5\ V$. At $t = t_0$, when this voltage is $v(t = t_0) = 3\ V$, the switch is closed, remaining in this new positivion indefinitely.

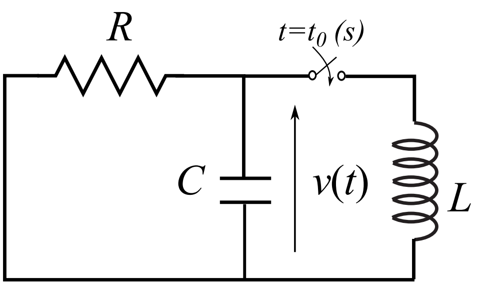


Figure 1

Data: $R = 0,5\Omega$; $C = 1F$; $L = 0,5H$

Determine:

a) The expression of $v(t)$ for $0 \leq t \leq t_0$ seg. Value of t_0.

b) The expression of $v(t)$ for $t \geq t_0$ seg.

c) The undamped natural frequency and the damping coefficient once the switch is closed. What is the type of damping of the circuit, according to the damping coefficient?

Result

a) $v(t) = 5 \cdot e^{-2t} \quad 0 \leq t \leq t_0,\ t_0 = 0.2554$ s.

b) $v(t) = 3\sqrt{2} \cdot e^{-(t-t_0)} \cdot \sin\left(t - t_0 + \frac{3\pi}{4}\right) \quad t > t_0.$

c) Underdamped. $\omega_n = \sqrt{2}$ rad/s, $\xi = \frac{1}{\sqrt{2}} < 1.$

Problem 1.13

In the circuit of the figure 1, where the current generator is a continuous source, the switcher is in position (1) since $t = -\infty$. At $t = 0$, the switcher changes to position (2), remaining in this position. Knowing that the voltage at the capacitor and the current through the inductor at $t = 0^-$ are respectively $V_C(0^-) = 10$ V and $I_L(0^-) = 5$ A. Calculate:

a) Values of I_g, R_g y R so that the initial conditions are fulfilled.

Supposing for the next section that $R = 2\ \Omega$,

b) Obtain the temporal expression for the voltage at the capacitor $v_C(t)$ for $t > 0$, indicating the type of circuit according to the damping ratio.

c) At $t = t_1$, when the voltage at the capacitor reaches the value of 2 V, the value of R is duplicated. Obtain the new value of the damping ratio and the corresponding type of circuit for $t > t_1$.

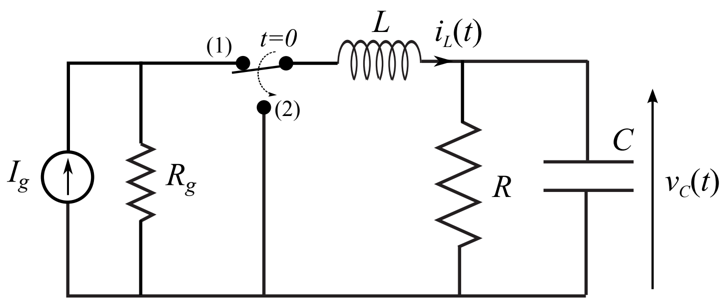


Figure 1

Data:

$$R_g = R; \quad L = 8\text{ H}; \quad C = \frac{1}{2}\text{ F}$$

Result

a) $\begin{cases} R = R_g = 2\ \Omega \\ I_g = 10\text{ AV} \end{cases}$

b) $v_C(t) = (5t + 10) \cdot e^{-\frac{t}{2}}; \qquad t > 0$, Circuit critically dumped.

c) Under damped circuit with $\xi = \frac{1}{2}$ y $\omega_n = \frac{1}{2}$ rad/s

Problem 1.14

For the circuit of the Figure 1 the switch is in position (1) from $t = -\infty\ s$. At $t = 0\ s$, the switch is in position (2). Compute the time-domain expression of the current $i_L(t)$ for $t > 0\ s$

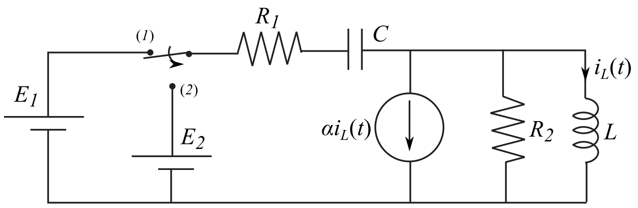
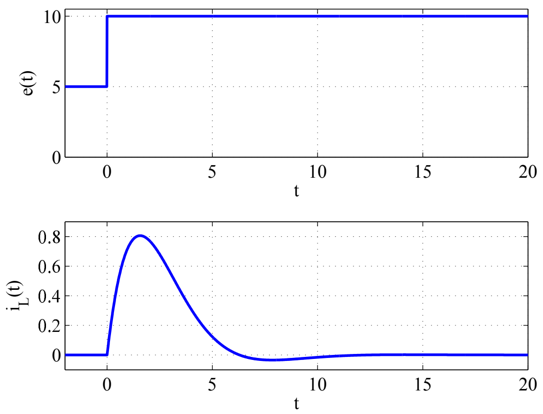


Figure 1

$$Data: \quad L = 2\ H \quad ; \quad C = 1\ F \quad ; \quad \alpha = 1 \quad ; \quad R_1 = R_2 = 1\ \Omega \quad ; \quad E_1 = 5\ V$$
$$E_2 = 10\ V$$

Result
$i_L(t) = \frac{5}{2} \cdot e^{-\frac{t}{2}} \cdot \sin\left(\frac{t}{2}\right)$



Problem 1.15

For the circuit of the Figure 1, the switch is in the position (1). At $t = 0\ s$ the switch conmmutes to position (2), remaining in this position. Determine the expression of $v_C(t)$ for all t.

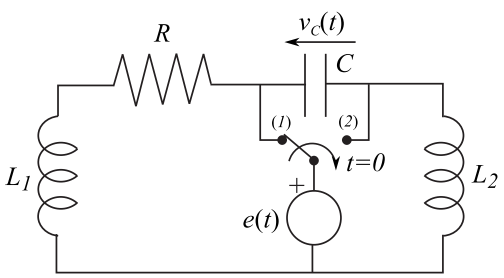
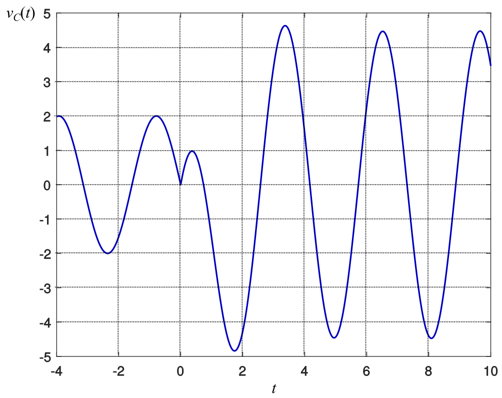


Figure 1

$$Data: \quad L_1 = 1\ H \quad ; \quad L_2 = 3\ H \quad ; \quad C = \frac{1}{2}\ F \quad ; \quad R = 2\Omega \quad ; \quad e(t) = 10 \cdot sin(2t)\ \text{V}$$

Result

$$v_C(t) = \begin{cases} 2 \cdot \text{sen}\,(2t + \pi)\ \text{V} & t < 0 \\ 4\left(\frac{\sqrt{5}}{2} \cdot sen(2t + 1.107) - \frac{5}{4} \cdot e^{-t} \cdot sen(t + 0.927)\right)\ \text{V} & t > 0 \end{cases}$$



Problem 1.16

For the circuit of the Figure 1(a) the switches S_1 and S_2 are in position (1) from $t = -\infty\ s$. At $t = 0\ s$ the switch S_1 commutes to position (2) remaining the switch (S_2) at position (1). Under these conditions we observe that the current in the inductor is the one shown in the Figure 1(b). Determine:

a) Value of the voltage of the source E_0 and of L.

b) Time-domain expression of the current in the inductor, $i_L(t)$, assuming that L is real, with and internal resistor $r = 8\ \Omega$. Indicate the type of damping of the circuit, and the damping coefficient.

c) Given the conditions of the previous section (real inductor L), at t_0 the switch S_2 commutes to position (2), while switch S_1 remains at position (2). Assuming that at t_0^- the circuit has reaches the stady-state, obtain the time-domain expression of the current in the inductor, $i_L(t)$, for $t > t_0$.

d) Compute the value of the current in the inductor passed 5 s since $t = t_0$.

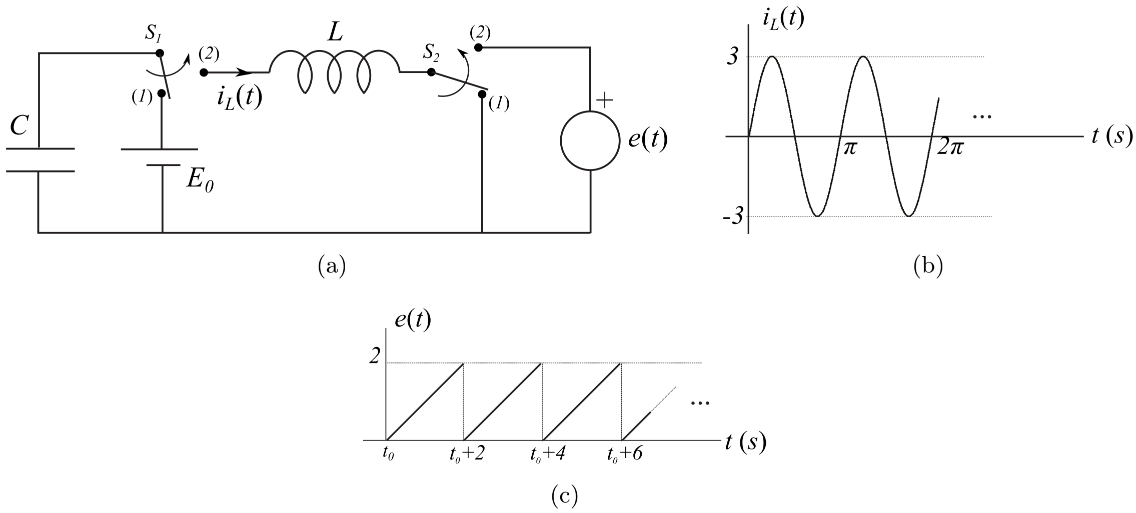


Figure 1

$$Data: \quad C = \frac{1}{8}\ F$$

Result

a) $L = 2\ H \quad ; \quad E_0 = 12\ V$

b) $i_L(t) = 6 \cdot t \cdot e^{-2t} \quad 0 < t < t_0$, Critically damped ($\xi = 1$)

c)
$$\begin{aligned} i_L(t) &= -\tfrac{1}{8} \cdot \left(1 - e^{-2(t-t_0)} - 2\,(t-t_0) \cdot e^{-2(t-t_0)}\right) \cdot u\,(t-t_0) \\ &+ \sum_{n=1}^{\infty} (t - t_0 - 2n) \cdot e^{-2(t-t_0-2n)} \cdot u\,(t - t_0 - 2n) \end{aligned}$$

d) $i_L(t = t_0 + 5) = 0.0178\ A$

UNILATERAL LAPLACE TRANSFORM

Nº	$F(s)$	$f(t) \quad t \geq 0$	Observations
1.-	1	$\delta(t)$	Delta function $t = 0$
2.-	$\frac{1}{s}$	$u(t)$	Step function $t = 0$
3.-	$\frac{1}{s^2}$	$tu(t)$	
4.-	$\frac{1}{s^n}$	$\frac{1}{(n-1)!} \cdot t^{n-1}$	n Positive integer
5.-	$\frac{1}{s} \cdot e^{-as}$	$u(t-a)$	Step function starting at $t = a$
6.-	$\frac{1}{s} \cdot \left(1 - e^{-as}\right)$	$u(t) - u(t-a)$	Rectangular pulse a
7.-	$\frac{1}{s+a}$	e^{-at}	
8.-	$\frac{1}{(s+a)^n}$	$\frac{1}{(n-1)!} \cdot t^{n-1} \cdot e^{-at}$	n Positive integer
9.-	$\frac{1}{s(s+a)}$	$\frac{1}{a} \cdot \left(1 - e^{-at}\right)$	
10.-	$\frac{1}{s(s+a)(s+b)}$	$\frac{1}{ab} \cdot \left(1 - \frac{b}{b-a} \cdot e^{-at} + \frac{a}{b-a} \cdot e^{-bt}\right)$	
11.-	$\frac{s+\alpha}{s(s+a)(s+b)}$	$\frac{1}{ab} \cdot \left(\alpha - \frac{b(\alpha-a)}{b-a} \cdot e^{-at} + \frac{a(\alpha-b)}{b-a} \cdot e^{-bt}\right)$	
12.-	$\frac{1}{(s+a)(s+b)}$	$\frac{1}{b-a} \cdot \left(e^{-at} - e^{-bt}\right)$	
13.-	$\frac{s}{(s+a)(s+b)}$	$\frac{1}{a-b} \cdot \left(a\, e^{-at} - b\, e^{-bt}\right)$	
14.-	$\frac{s+\alpha}{(s+a)(s+b)}$	$\frac{1}{b-a} \cdot \left[(\alpha-a)\, e^{-at} - (\alpha-b)\, e^{-bt}\right]$	
15.-	$\frac{s+\alpha}{(s+a)^2}$	$[(\alpha-a)\, t + 1] \cdot e^{-at}$	
16.-	$\frac{1}{(s+a)(s+b)(s+c)}$	$\frac{e^{-at}}{(b-a)(c-a)} + \frac{e^{-bt}}{(c-b)(a-b)} + \frac{e^{-ct}}{(a-c)(b-c)}$	
17.-	$\frac{s+\alpha}{(s+a)(s+b)(s+c)}$	$\frac{(\alpha-a) \cdot e^{-at}}{(b-a)(c-a)} + \frac{(\alpha-b) \cdot e^{-bt}}{(c-b)(a-b)} + \frac{(\alpha-c) \cdot e^{-ct}}{(a-c)(b-c)}$	
18.-	$\frac{\omega}{s^2+\omega^2}$	$\sin(\omega\, t)$	
19.-	$\frac{s}{s^2+\omega^2}$	$\cos(\omega\, t)$	
20.-	$\frac{s+\alpha}{s^2+\omega^2}$	$\frac{\sqrt{\alpha^2+\omega^2}}{\omega} \cdot \sin(\omega\, t + \phi) \quad ; \quad \phi = \text{atan}\left(\frac{\omega}{\alpha}\right)$	
21.-	$\frac{s \cdot \sin(\phi) + \omega \cdot \cos(\phi)}{s^2+\omega^2}$	$\sin(\omega\, t + \phi)$	
22.-	$\frac{s \cdot \cos(\phi) - \omega \cdot \sin(\phi)}{s^2+\omega^2}$	$\cos(\omega\, t + \phi)$	

Nº	$F(s)$	$f(t) \qquad t \geq 0$	**Observations**
23.-	$\dfrac{1}{s\,(s^2+\omega^2)}$	$\dfrac{1}{\omega^2}\cdot[1-\cos(\omega\, t)]$	
24.-	$\dfrac{s+\alpha}{s\,(s^2+\omega^2)}$	$\dfrac{\alpha}{\omega^2}-\dfrac{\sqrt{\alpha^2+\omega^2}}{\omega^2}\cdot\cos(\omega\, t+\phi) \quad ; \quad \phi=\text{atan}\left(\dfrac{\omega}{\alpha}\right)$	
25.-	$\dfrac{1}{(s+a)\,(s^2+\omega^2)}$	$\dfrac{1}{a^2+\omega^2}\cdot e^{-at}-\dfrac{1}{\omega\sqrt{a^2+\omega^2}}\cdot\sin(\omega\, t+\phi) \quad ; \quad \phi=\text{atan}\left(\dfrac{\omega}{-a}\right)$	
26.-	$\dfrac{s^2}{(s+a)\,(s^2+\omega^2)}$	$\dfrac{a^2}{a^2+\omega^2}\cdot e^{-at}+\dfrac{\omega}{\sqrt{a^2+\omega^2}}\cdot\sin(\omega\, t+\phi) \quad ; \quad \phi=\text{atan}\left(\dfrac{\omega}{-a}\right)$	
27.-	$\dfrac{1}{s^2+2\xi\omega_n s+\omega_n^2}$	$\dfrac{1}{\omega_n\sqrt{1-\xi^2}}\cdot e^{-\xi\omega_n t}\cdot\sin\left(\omega_n\sqrt{1-\xi^2}\; t\right)$	$0<\xi<1$
28.-	$\dfrac{1}{(s+a)^2+b^2}$	$\dfrac{1}{b}\cdot e^{-at}\cdot\sin(b\, t)$	
29.-	$\dfrac{s+a}{(s+a)^2+b^2}$	$e^{-at}\cdot\cos(b\, t)$	
30.-	$\dfrac{s+\alpha}{(s+a)^2+b^2}$	$\dfrac{\sqrt{(\alpha-a)^2+b^2}}{b}\cdot e^{-at}\cdot\sin(b\, t+\phi) \quad ; \quad \phi=\text{atan}\left(\dfrac{b}{\alpha-a}\right)$	
31.-	$\dfrac{1}{s\,(s^2+2\xi\omega_n s+\omega_n^2)}$	$\dfrac{1}{\omega_n^2}-\dfrac{1}{\omega_n^2\sqrt{1-\xi^2}}\cdot e^{-\omega_n\xi t}\cdot\sin\left(\omega_n\sqrt{1-\xi^2}\; t+\phi\right) \quad ; \quad \phi=\text{acos}(\xi)$	$0<\xi<1$
32.-	$\dfrac{1}{s\,[(s+a)^2+b^2]}$	$\dfrac{1}{a^2+b^2}+\dfrac{1}{b\sqrt{a^2+b^2}}e^{-at}\cdot\sin(b\, t-\phi) \quad ; \quad \phi=\text{atan}\left(\dfrac{b}{-a}\right)$	
33.-	$\dfrac{s+\alpha}{s\,[(s+a)^2+b^2]}$	$\dfrac{\alpha}{a^2+b^2}+\dfrac{1}{b}\cdot\sqrt{\dfrac{(\alpha-a)^2+b^2}{a^2+b^2}}\cdot e^{-at}\cdot\sin(b\, t+\phi)$ $\phi=\text{atan}\left(\dfrac{b}{\alpha-a}\right)-\text{atan}\left(\dfrac{b}{-a}\right)$	
34.-	$\dfrac{1}{(s+c)\,[(s+a)^2+b^2]}$	$\dfrac{1}{(c-a)^2+b^2}\cdot e^{-ct}+\dfrac{1}{b\sqrt{(c-a)^2+b^2}}\cdot e^{-at}\cdot\sin(b\, t-\phi)$ $\phi=\text{atan}\left(\dfrac{b}{c-a}\right)$	
35.-	$\dfrac{1}{s(s+c)\,[(s+a)^2+b^2]}$	$\dfrac{1}{c\,(a^2+b^2)}-\dfrac{e^{-ct}}{c\,[(c-a)^2+b^2]}+\dfrac{e^{-at}\cdot\sin(b\, t-\phi)}{b\sqrt{a^2+b^2}\sqrt{(c-a)^2+b^2}}$ $\phi=\text{atan}\left(\dfrac{b}{-a}\right)+\text{atan}\left(\dfrac{b}{c-a}\right)$	
36.-	$\dfrac{s+\alpha}{s(s+c)\,[(s+a)^2+b^2]}$	$\dfrac{\alpha}{c\,(a^2+b^2)}-\dfrac{(c-\alpha)\cdot e^{-ct}}{c\,[(c-a)^2+b^2]}+\dfrac{\sqrt{(\alpha-a)^2+b^2}\cdot e^{-at}\cdot\sin(b\, t+\phi)}{b\sqrt{a^2+b^2}\sqrt{(c-a)^2+b^2}}$ $\phi=\text{atan}\left(\dfrac{b}{\alpha-a}\right)-\text{atan}\left(\dfrac{b}{-a}\right)-\text{atan}\left(\dfrac{b}{c-a}\right)$	
37.-	$\dfrac{1}{s^2(s+a)}$	$\dfrac{1}{a^2}\cdot\left(at-1+e^{-at}\right)$	
38.-	$\dfrac{1}{s(s+a)^2}$	$\dfrac{1}{a^2}\cdot\left(1-e^{-at}-at\, e^{-at}\right)$	
39.-	$\dfrac{s+\alpha}{s(s+a)^2}$	$\dfrac{1}{a^2}\cdot\left[\alpha-\alpha\, e^{-at}+a(a-\alpha)t\, e^{-at}\right]$	

Nº	$F(s)$	$f(t) \qquad t \geq 0$	**Observations**
40.-	$\dfrac{s^2+\alpha_1 s+\alpha_0}{s\,(s+a)\,(s+b)}$	$\dfrac{\alpha_0}{ab}+\dfrac{a^2-\alpha_1 a+\alpha_0}{a(a-b)}\cdot e^{-at}-\dfrac{b^2-\alpha_1 b+\alpha_0}{b(a-b)}\cdot e^{-bt}$	
41.-	$\dfrac{s^2+\alpha_1 s+\alpha_0}{s\,[(s+a)^2+b^2]}$	$\dfrac{\alpha_0}{a^2+b^2}+$ $+\dfrac{\sqrt{a^2-b^2-\alpha_1 a+\alpha_0)^2+b^2(\alpha_1-2a)^2}}{b(a^2+b^2)}\cdot e^{-at}\cdot\sin(bt+\phi)$ $\phi=\text{atan}\left(\dfrac{b(\alpha_1-2a)}{a^2-b^2-\alpha_1 a+\alpha_0}\right)-\text{atan}\left(\dfrac{b}{-a}\right)$	
42.-	$\dfrac{1}{(s^2+\omega^2)\,[(s+a)^2+b^2]}$	$\dfrac{\frac{1}{\omega}\cdot\sin(\omega t+\phi_1)+\frac{1}{b}\cdot e^{-at}\cdot\sin(bt+\phi_2)}{\sqrt{4a^2\omega^2+(a^2+b^2-\omega^2)^2}}$ $\phi_1=\text{atan}\left(\dfrac{-2a\omega}{a^2+b^2-\omega^2}\right)\quad;\quad\phi_2=\text{atan}\left(\dfrac{2ab}{a^2-b^2+\omega^2}\right)$	
43.-	$\dfrac{s+\alpha}{(s^2+\omega^2)\,[(s+a)^2+b^2]}$	$\dfrac{1}{\omega}\cdot\sqrt{\dfrac{\alpha^2+\omega^2}{(2a\omega)^2+(a^2+b^2-\omega^2)^2}}\cdot\sin(\omega t+\phi_1)+$ $+\dfrac{1}{b}\cdot\sqrt{\dfrac{(\alpha-a)^2+b^2}{(2a\omega)^2+(a^2+b^2-\omega^2)^2}}\cdot e^{-at}\cdot\sin(bt+\phi_2)$ $\phi_1=\text{atan}\left(\dfrac{\omega}{\alpha}\right)-\text{atan}\left(\dfrac{2a\omega}{a^2+b^2+\omega^2}\right)$ $\phi_2=\text{atan}\left(\dfrac{b}{\alpha-a}\right)+\text{atan}\left(\dfrac{2ab}{a^2-b^2+\omega^2}\right)$	
44.-	$\dfrac{s+\alpha}{s^2\,[(s+a)^2+b^2]}$	$\dfrac{1}{a^2+b^2}\cdot\left(\alpha t+1-\dfrac{2\alpha a}{a^2+b^2}\right)+\dfrac{\sqrt{b^2+(\alpha-a)^2}}{b(a^2+b^2)}\cdot e^{-at}\cdot\sin(bt+\phi)$ $\phi=2\text{atan}\left(\dfrac{b}{a}\right)+\text{atan}\left(\dfrac{b}{\alpha-a}\right)$	
45.-	$\dfrac{s^2+\alpha_1 s+\alpha_0}{s^2(s+a)(s+b)}$	$\dfrac{\alpha_1+\alpha_0 t}{ab}-\dfrac{\alpha_0(a+b)}{(ab)^2}-\dfrac{1}{a-b}\cdot\left(1-\dfrac{\alpha_1}{a}+\dfrac{\alpha_0}{a^2}\right)\cdot e^{-at}$ $+\dfrac{1}{a-b}\cdot\left(1-\dfrac{\alpha_1}{b}+\dfrac{\alpha_0}{b^2}\right)\cdot e^{-bt}$	