PROBLEMS OF CHAPTER 2: Two-port networks.

February 22, 2018

Problem 2.1

The two-port network Q of figure 1(a) is connected in the circuit shown in Figure 1(b). Obtain:

a) The family of z-parameters for the two-port network.

b) The power absorbed by the load Z_R.

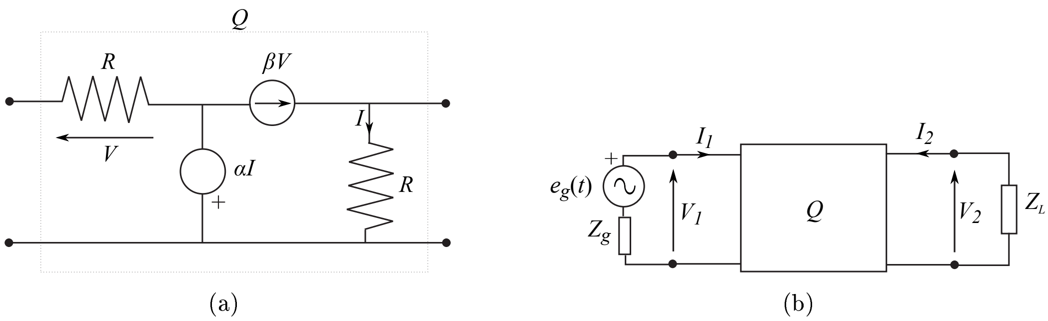


Figure 1

$$Data: \quad R = 1\,\Omega \quad ; \quad \alpha = 1\,\Omega \quad ; \quad \beta = 1\,\Omega^{-1}$$
$$Z_g = 1\,\Omega \quad ; \quad Z_R = 1 + j\,\Omega \quad ; \quad e_g(t) = 4\,sin(t)\,V$$

Result

a) $\begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}$

b) $P_{Z_R} = \frac{4}{5} \cdot W$

Problem 2.2

a) On the quadripole Q_1 we obtain the measurements shown in figure 1(a) and 1(b), obtaining the following results:

$$\text{Circuit (a)}: \begin{cases} i_1(t) = 2\,\sin(\omega t)\ \text{A} \\ i_2(t) = 2\,\sin\left(\omega t - \dfrac{\pi}{2}\right)\ \text{A} \end{cases} \quad ; \quad \text{Circuit (b)}: \begin{cases} i_3(t) = 2\,\sin\left(\omega t - \dfrac{\pi}{2}\right)\ \text{A} \\ v_4(t) = \sin\left(\omega t + \dfrac{\pi}{2}\right)\ \text{V} \end{cases}$$

Obtain the h-parameters of the quadripole.

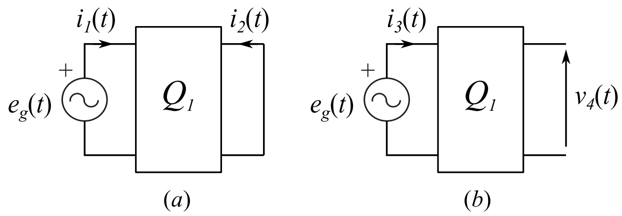


Figure 1

Data: $e_g(t) = 4\sin(\omega t)$ V

b) Obtain the T-parameters of the quadripole with the structure shown in figure 2.

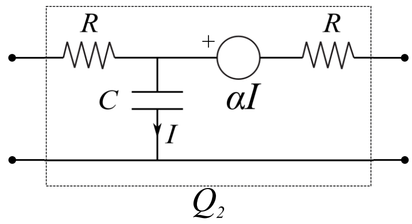


Figure 2

Data: $R = \alpha = \dfrac{1}{2}\ \Omega$; $Z_C = -\dfrac{1}{2}j\ \Omega$.

Result

a) $(h_{Q_1}) = \begin{pmatrix} 2\ \Omega & 4-4j \\ -j & -2j\ \Omega^{-1} \end{pmatrix}$

b) $(T_{Q_2}) = \begin{pmatrix} j & \dfrac{1+j}{2}\ \Omega \\ (-1+j)\ \Omega^{-1} & \dfrac{1+j}{2} \end{pmatrix}$

Problem 2.3

The quadripole Q_1 of the associated quadripoles shown in figure 2 is RLC and measurements on this quadripole are shown in figure 1(a) and (b). The quadripole Q_2 is symmetrical and two of his parameters are $h_{11} = 5\ \Omega$ and $h_{12} = 1$.

a) Obtain a family of parameters that characterizes the association of the quadripoles Q_1 and Q_2 as it is shown in Figure 2.

b) Obtain the impedance Z_e.

c) Obtain $i_2(t)$.

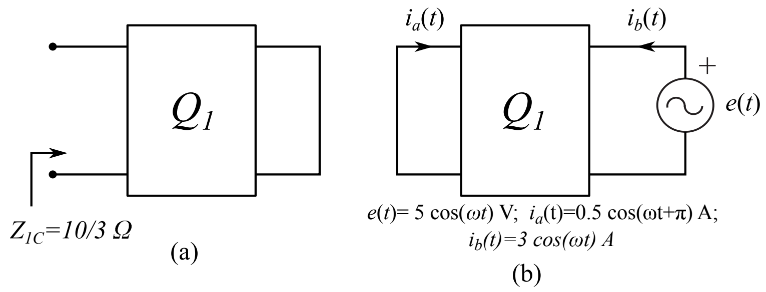


Figure 1

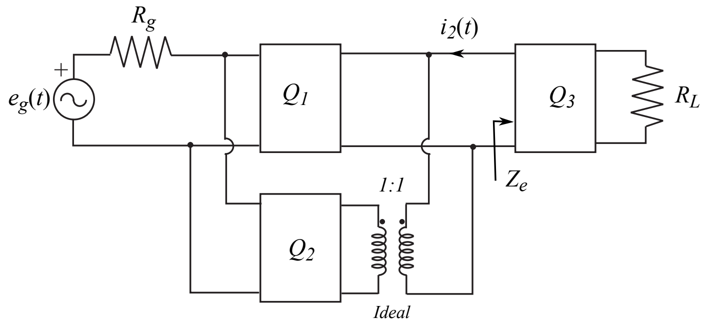


Figure 2

DATOS: $R_g = 10\ \Omega$; $R_L = 50\ \Omega$; $e_g(t) = 10\ cos(\omega_0 t)\ V$

$$Q_3 = \begin{pmatrix} A = \dfrac{\sqrt{2}}{2} & B = 25\sqrt{2}j\ \Omega \\ C = \dfrac{\sqrt{2}}{100}j\ \Omega^{-1} & D = \dfrac{\sqrt{2}}{2} \end{pmatrix}$$

Result

a) $\begin{pmatrix} y_{11}^T & y_{12}^T \\ y_{21}^T & y_{22}^T \end{pmatrix} = (y_{Q_1}) + (y_{Q_2}) = \begin{pmatrix} \frac{3}{10} & -\frac{1}{10} \\ -\frac{1}{10} & \frac{3}{5} \end{pmatrix} + \begin{pmatrix} \frac{1}{5} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{1}{5} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & -\frac{3}{10} \\ -\frac{3}{10} & \frac{4}{5} \end{pmatrix}$

b) $Z_e = 50\ \Omega$

c) $i_2(t) = \frac{1}{67} \cdot cos(\omega_0 t + \pi)\ A$

Problem 2.4

In figure 1(a) an association of two quadripoles, Q_A and Q_B, is shown and their circulation currents are zero. The quadripole Q_A is symmetrical and some measurements on Q_A are shown in figure 1(b). The quadripole Q_B is RLC and for this quadripole it is known $Z_{1,O} = \dfrac{4}{5}\ \Omega$, $Z_{1,S} = 0\ \Omega$, $Z_{2,O} = \dfrac{1}{5}\ \Omega$ and the phase of the inverse voltage gain with input port in open circuit is zero. Calculate a family of parameters that characterizes the association of the two quadripoles as it is shown in Figure 1(a)

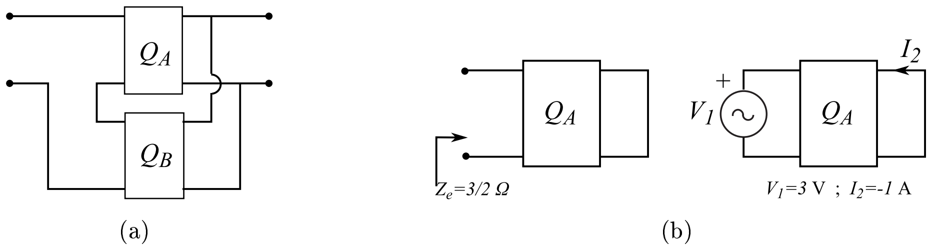


Figure 1

Result

$$(h_{Q_T}) = (h_{Q_A}) + (h_{Q_B}) = \begin{pmatrix} \frac{3}{2}\ \Omega & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2}\ \Omega^{-1} \end{pmatrix} + \begin{pmatrix} 0\ \Omega & 2 \\ -2 & 5\ \Omega^{-1} \end{pmatrix} = \begin{pmatrix} \frac{3}{2}\ \Omega & \frac{5}{2} \\ -\frac{5}{2} & \frac{11}{2}\ \Omega^{-1} \end{pmatrix}$$

Problema 2.5

In the circuit of the figure 1(a) the currents $i_1(t)$ and $i_2(t)$ are known. Furthermore, the internal structure of the quadripole Q is the one shown in figure 1(b).

a) Calculate the values of R_1 and R_2.

b) Obtain $i_1(t)$ if $Z_L \to \infty$.

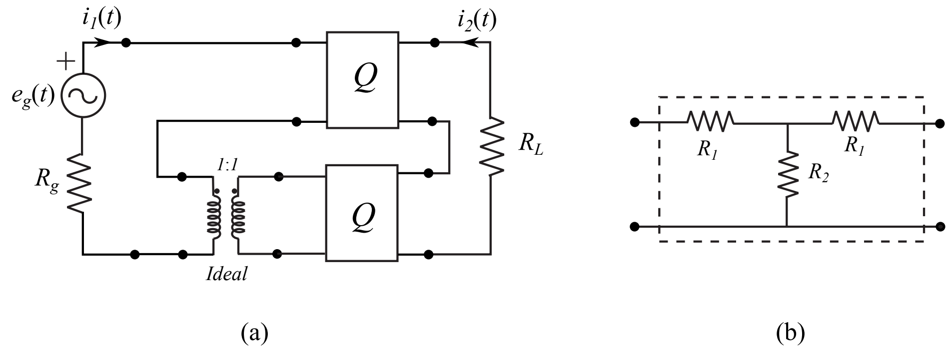


Figure 1

$$\begin{aligned} DATOS: \quad & e_g(t) = 4 \cdot \cos(\omega t)\ \text{V.} \quad ; \quad i_1(t) = \cos(\omega t)\ A \quad ; \quad i_2(t) = \tfrac{1}{4} \cdot \cos(\omega t + \pi)\ \text{A} \\ & R_g = 1\ \Omega \quad ; \quad R_L = 2\ \Omega \end{aligned}$$

Result

a) $R_1 = 1\ \Omega$; $R_2 = \frac{2}{3}\ \Omega$

b) $i_1(t) = \dfrac{12}{13} \cos(\omega t)\ A$

Problem 2.6

For a symmetrical quadripole, Q, we have performed the measurements shown in figure 1(a). Calculate a family of parameters to characterize the association shown in figure 1(b) of two of these quadripoles.

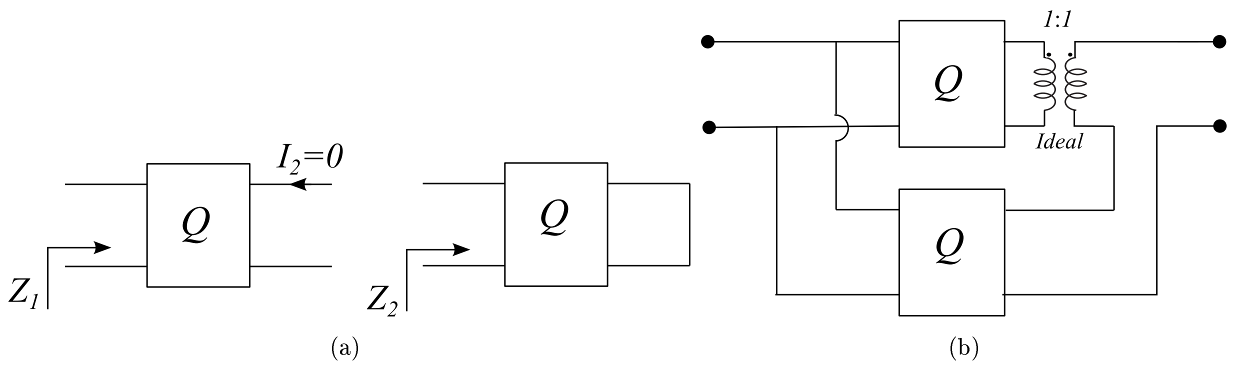


Figure 1

Data: $Z_1 = 2\ \Omega$ and $Z_2 = 4\ \Omega$.

Result

$$(g_{Q_T}) = \begin{pmatrix} 1\ \Omega^{-1} & \pm 2j \\ \mp 2j & 8\ \Omega \end{pmatrix}$$

Problem 2.7

For the associated quadripoles shown in Figure 1, calculate:

a) The most adequate family of parameters for the association of quadripoles Q, which is shown in Figure 2.

b) The time-domain expression of the current that goes through the source $e_g(t)$.

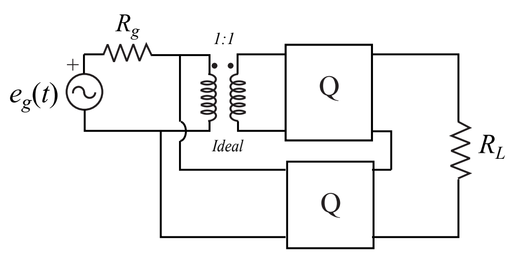


Figure 1

$$DATA:\ \ e_g(t) = 17\ sen\,(t)\ V \ \ ; \ \ R_g = \frac{2}{5}\ \Omega \ \ ; \ \ R_L = \frac{15}{2}\ \Omega$$

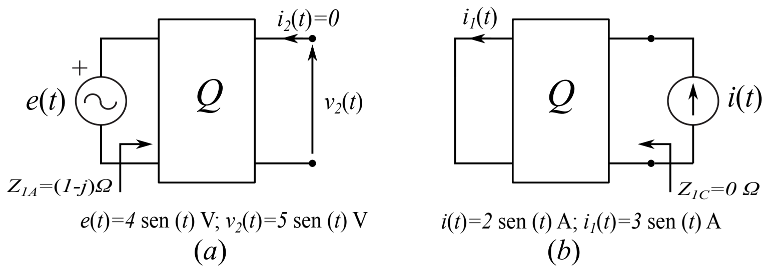


Figure 2

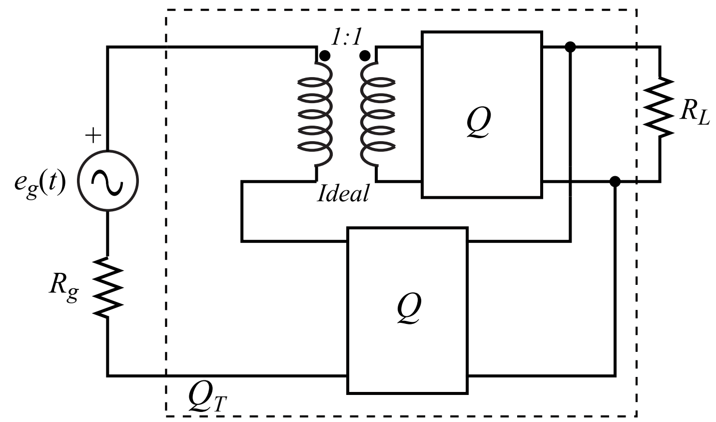
Result

a) $(g_Q) = \begin{pmatrix} \frac{1}{2} \cdot (1+j)\ \Omega^{-1} & -\frac{3}{2} \\ \frac{5}{4} & 0\ \Omega \end{pmatrix}$

b) $i_1(t) = 5\sqrt{17}\ \text{sen}\,(t + 0.245)\ A$

Problem 2.8

In the circuit of the figure 1, we know the h-parameters of the two-port network (Q_T). Note that this quadripole Q_T represents the association of two identical quadripoles (Q). However, the internal structure of the quadripole Q is not known, but it can be one of the structures shown in figure 2(a) or (b).



$$(h)_{Q_T} = \begin{pmatrix} 5\ \Omega & 1 \\ -\frac{1}{2} & \frac{1}{2}\ \Omega^{-1} \end{pmatrix}$$

Figure 1

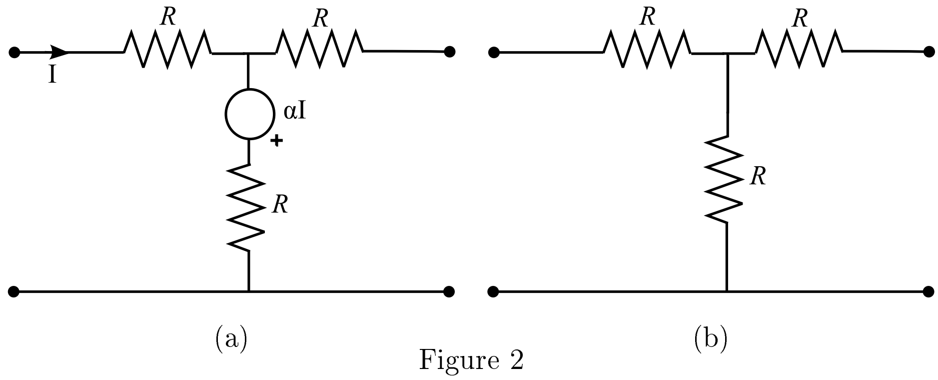


Figure 2

Answer the following questions:

a) Reason and choose, without performing any calculation, which of the two circuits (a) or (b) shown in figure 2 corresponds to the quadripole Q of the figure 1.

b) Calculate the values of the elements of the quadripole Q: (R and α if the selected Q is (a), or R if the selected Q is (b)).

c) Calculate the power dissipated by R_L.

$$\textbf{Data}: \quad e_g(t) = 13\cos(\omega t)\ V; \quad R_g = 1\ \Omega; \quad R_L = 2\ \Omega$$

Result

a) (a).

b) $R = 2\ \Omega$; $\alpha = 1\ \Omega$

c) $P_{R_L} = \dfrac{1}{4}$ W

Problem 2.9

We want to associate the two quadripoles Q_A and Q_B, as it is shown in figure 1. For the quadripole cuadripolo Q_A the $z-$parameters are known. From the quadripole Q_B we know that it is reciprocal and the measurements shown in figure 2.

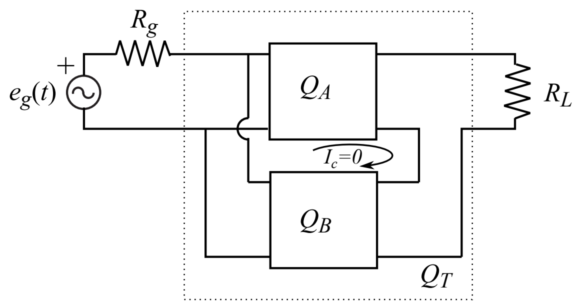


Figure 1

$$\text{DATOS: } R_g = R_L = 2\,\Omega\,; e_g(t) = \cos(t)\,V, \quad (z_{Q_A}) = \begin{pmatrix} -4j\ \Omega & -2j\ \Omega \\ -2j\ \Omega & 0\ \Omega \end{pmatrix}.$$

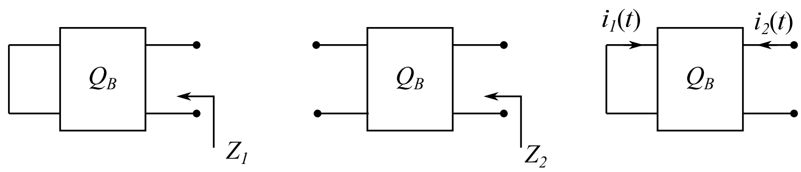


Figure 2

DATA: $Z_1 = 1\,\Omega$; $\quad Z_2 = 2\ \Omega$; $\quad i_1(t) = \cos(t)\,A$; $\quad i_2(t) = 2\ \cos(t)\,A$

a) Obtain the adequate family of parameters for Q_A y Q_B so that they can be association to the Q_T.

b) The power dissipoated by the resistor $R_L$in the circuit of the figure 1.

Result

a) $(g_{Q_A}) = \begin{pmatrix} \frac{j}{4}\ \Omega^{-1} & -\frac{1}{2} \\ \frac{1}{2} & j\ \Omega \end{pmatrix}$, $(g_{Q_B}) = \begin{pmatrix} \frac{1}{4}\ \Omega^{-1} & \frac{1}{2} \\ -\frac{1}{2} & 1\ \Omega \end{pmatrix}$

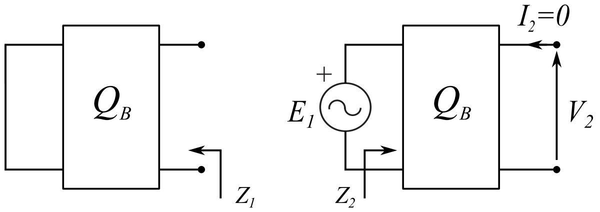
b) $P_{R_L} = 0$ W

Problem 2.10

For the association of quadripoles shown in Figure 2 we know that the circulation currents are zero ($I_c = 0$)

a) Obtain a family of parameters to characterize the RLC quadripole (Q_B) on which the measurements shown in figure 1 have been done.

b) Obtain the equivalent impedance between the points C and D to the left (Z_S) in the circuit of the figure 2.



Datos: $\begin{cases} Z_1 = 2\ \Omega \\ Z_2 = \frac{2}{3}\ \Omega \\ V_2 = 2E_1 \end{cases}$

Figure 1

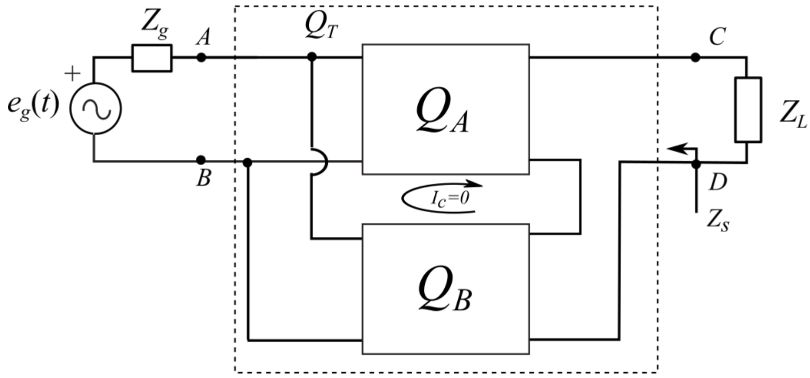


Figure 2

DATA: $(z_{Q_A}) = \begin{pmatrix} 2\ \Omega & 2\ \Omega \\ 2\ \Omega & (2-2j)\ \Omega \end{pmatrix}$; $Z_g = 1\ \Omega$

Result

a) $(g_{Q_A}) = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} = \begin{pmatrix} \frac{1}{2}\ \Omega^{-1} & -1 \\ 1 & -2j\ \Omega \end{pmatrix}$; $(g_{Q_B}) = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} = \begin{pmatrix} \frac{3}{2}\ \Omega^{-1} & -2 \\ 2 & 2\ \Omega \end{pmatrix}$

b) $Z_S = (5-2j)\ \Omega$

Problem 2.11

For the associated quadripoles Q_A and Q_B shown in figure 3 it is known that the circulation currents are zero and also that:

- The quadripole Q_A is RLC and on measurements on this quadripole are shown in figure 1(a) y 1(b)

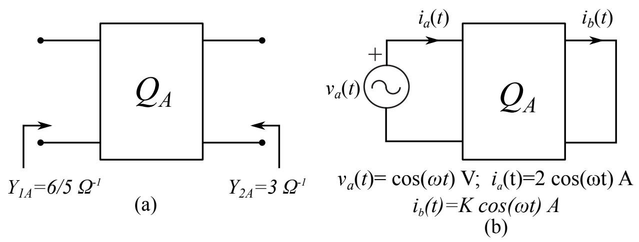


Figure 1

- The quadripole Q_B is symmetrical and measurements on this quadripole are shown in figure 2(a) y 2(b).

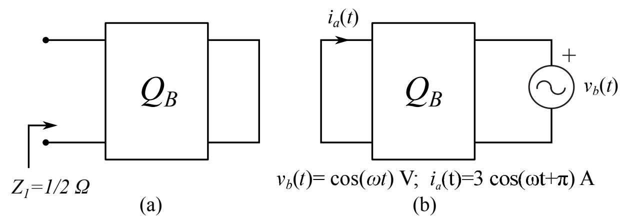


Figure 2

Optain:

a) A family of parameters that characterizes the association of the quadripoles Q_A y Q_B shown in figure 3.

b) Power delivered by the real generator ($e_g(t)$, R_g).

c) Temporal expresssion of the current $i_x(t)$.

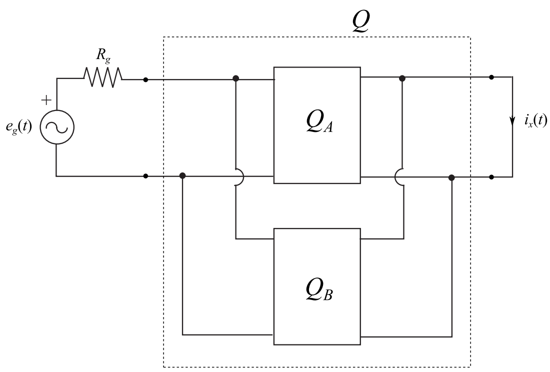


Figure 3

$$\text{DATA:} \quad e_g(t) = 4\cos(\omega t)\ \text{V} \quad ; \quad R_g = \tfrac{1}{4}\ \Omega$$

Result

a) $(y_{Q_T}) = (y_{Q_A}) + (y_{Q_B}) = \begin{pmatrix} 2\ \Omega^{-1} & -2\ \Omega^{-1} \\ -2\ \Omega^{-1} & 5\ \Omega^{-1} \end{pmatrix} + \begin{pmatrix} 2\ \Omega^{-1} & -3\ \Omega^{-1} \\ -3\ \Omega^{-1} & 2\ \Omega^{-1} \end{pmatrix} = \begin{pmatrix} 4\ \Omega^{-1} & -5\ \Omega^{-1} \\ -5\ \Omega^{-1} & 7\ \Omega^{-1} \end{pmatrix}$

b) $P_{E_g,R_g} = 8$ W

c) $i_x(t) = 10\cos(\omega t)$ A