PROBLEMS OF CHAPTER 3: Transfer of power. Image Parameters.

March 13, 2017

Problem 3.1

For the circuit in Figure 1, it is known that the real source $(e_g(t), R_g)$ delivers the maximum power to the network LC. If we know the measurements of the voltmeter and ammeter are $V_v = \sqrt{5}$ V_{ef} and $I_A = 2$ A_{ef}, respectively, obtain:

a) The power absorbed by resistor R.

b) Value of C.

c) Equivalent impedance from the output of the LC network.

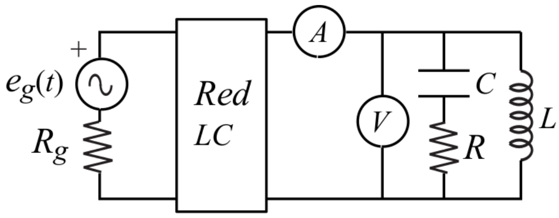

Figure 1

Data: $e_g(t) = 4\cos(t)$ V ; $R_g = 1\,\Omega$; $R = 2\,\Omega$

Result

a) $P_R = 2$ W.

b) $C = 1$ F.

c) $Z_S = \dfrac{1-2j}{2}\ \Omega$

Problem 3.2

For the circuit in Figure 1 there is impedance matching in section CD. We know that the network 2 has a power transmission loss of $\alpha_T = 6$ dBs. Determine:

a) Value of voltage V_{AB}.

b) Power in impedance Z_L.

c) Transmission and insertion loss for the set of two networks.

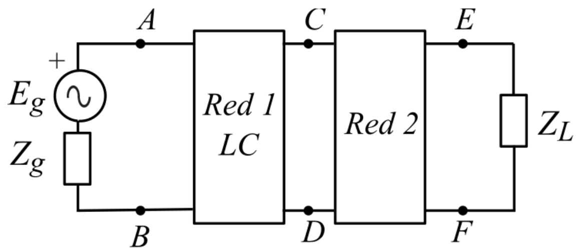

Figure 1

Data: $E_g = (1 + j)$ V ; $Z_g = (2 - 2j)\ \Omega$; $Z_L = (2 + 5j)\ \Omega$.

Result

a) $V_{AB} = j$ V.

b) $P_{Z_L} = \dfrac{1}{32}$ W

c) $\alpha_{Total} = \alpha_T = 6$ dBs, $\alpha_I = 4.08$ dBs

Problem 3.3

In the circuit of Figure 1 we have impedance matching in section CD. Network 2 introduces a power transmission loss of $\alpha_T = 3$ dBs and the measurement of the voltmeter is of $\sqrt{2}\ V_{ef}$. Determine:

a) Power of the impedance Z_L.

b) Value of impedance Z_C.

c) Transmission and insertion loss for the set of two networks.

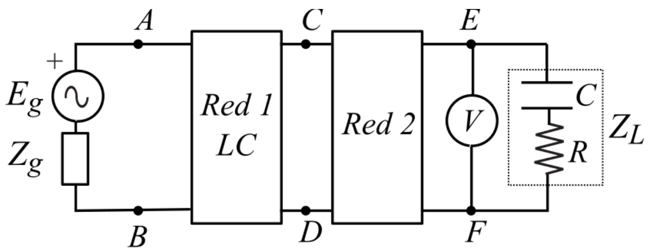

Figure 1

Data: $E_g = -8j$ V ; $Z_g = (4 - 4j)\ \Omega$; $R = 1\ \Omega$.

Result

a) $P_{Z_L} = 1$ W.

b) $Z_C = -j\ \Omega$.

c) $\alpha_T = 3$ dBs, $\alpha_I = -1.94$ dBs

Problem 3.4

In the circuit of Figure 1, we have impedance matching in section AB, and the voltmeter measures a voltage of 5 V_{ef}. Obtain:

a) Input impedance for network 2, Z_{CD}.

b) Power absorbed in R_2.

c) The value of inductor L_2.

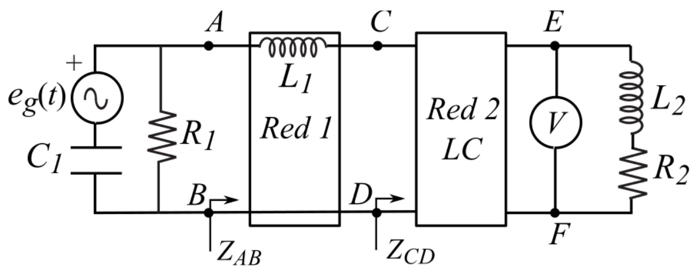

Figure 1

Data: $e_g(t) = 4\ \cos(t)$ V ; $R_1 = 2\ \Omega$; $R_2 = 9\ \Omega$; $L_1 = 2$ H ; $C_1 = \frac{1}{2}$ F.

Result

a) $Z_{CD} = (1 - j)\ \Omega$.

b) $P_{R_2} = 1$ W.

c) $L_2 =$ H

Problem 3.5

For the circuit shown in Figure 1(a), we know that the quadripoles have the internal structure shown in Figure 1(b). Compute:

a) The image parameters (Z_0 y γ) of the quadripoles.

b) Value for a, in order to guarantee that the power of the source $i_g(t)$ is 0 W.

c) Power for impedance Z_L.

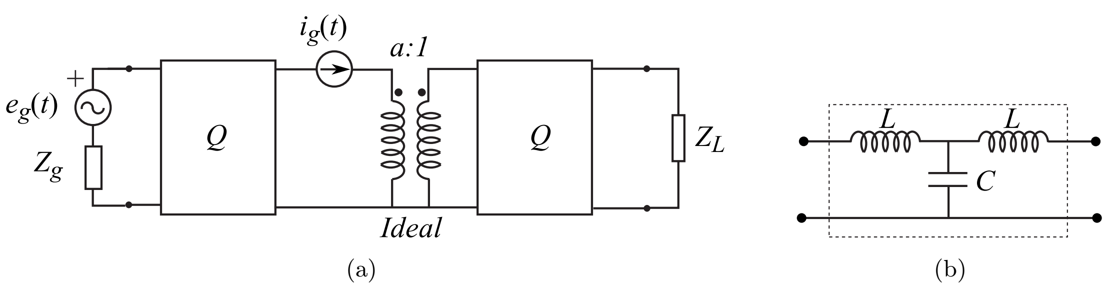

Figure 1

$$Data: \quad i_g(t) = \frac{2}{\sqrt{3}}\sin(\omega t)\ A \quad ; \quad \omega L = 5\ \Omega \quad ; \quad \frac{1}{\omega C} = 10\ \Omega$$
$$e_g(t) = 20\sin\left(\omega t + \frac{\pi}{3}\right)\ V \quad ; \quad Z_g = Z_L = 5\sqrt{3}\ \Omega$$

Result

a) $Z_0 = 5\sqrt{3}\ \Omega$; $\gamma = j\frac{\pi}{3}$

b) $a = 1$

c) $P_{Z_L} = \frac{10\sqrt{3}}{3}\ W$

Problem 3.6

For the circuit in Figure 1(a), networks Q_A and Q_B have the same impedance Z_0. If quadripole Q_A has the internal structure shown in Figure 1(b), determine:

a) Image parameters of quadripole Q_A.

b) Power absorbed by the load.

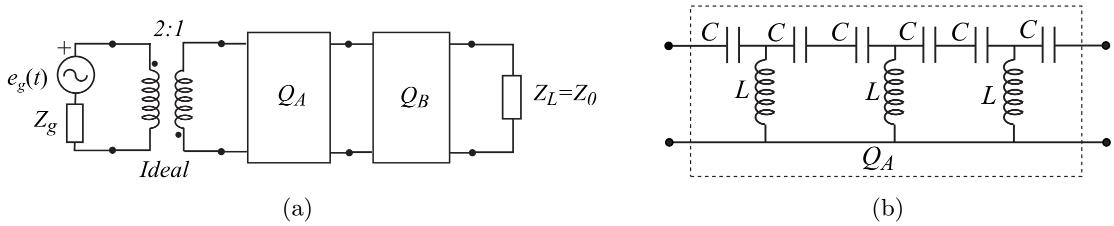

Figure 1

$$Data: \quad e_g(t) = 24\ sin(\omega t)\ V \quad ; \quad Z_g = 12\ \Omega$$
$$Q_A\ \left\{\omega L = 5\ \Omega\ ;\ \frac{1}{\omega C} = 1\ \Omega\right\} \quad ; \quad Q_B\ \left\{Z_0 = Z_0^A\ ;\ \gamma = \ln(2)\ \text{Np}\right\}$$

Result

a) $Z_{01} = 3\ \Omega$; $\gamma_A = 1.9305j$

b) $P_{Z_L} = 1.5\ W$

Problem 3.7

For the circuit shown in Figure 1(a) we know the image parameters of quadripole Q_A and the internal structure of two-port network Q_B, see Figure 1(b).

a) Obtain the value of the impedance Z in order to guarantee that the source (E_g, Z_g) delivers the maximum power.

b) Find the power absorbed by the impedance Z obtained in the previous section, as well as the time-domain expression of current $i_Z(t)$.

c) Justify why the insertion loss of quadripole Q_B is 0.

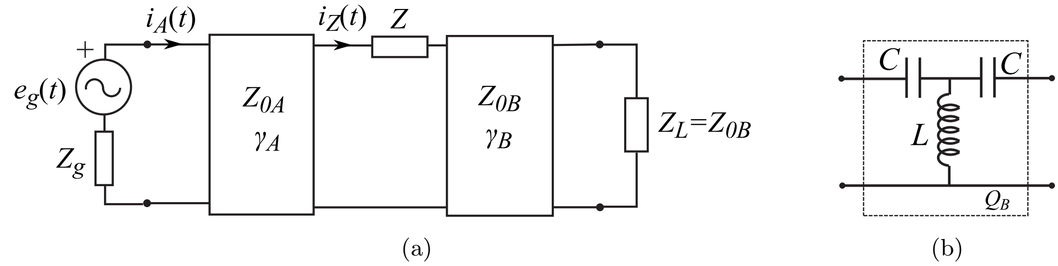

Figure 1

$$\begin{aligned} Data: \quad & e_g(t) = 6\, sin\,(\omega t)\ V \\ & Z_g = 6\ \Omega \qquad ; \quad Z_L = Z_{0B} \\ & Q_A\ \{Z_{0A} = 6\ \Omega\ ;\ \gamma = \ln(2)\ \text{Np}\ \}\quad ;\quad Q_B\ \left\{\omega L = 5\ \Omega\ ;\ \frac{1}{\omega C} = 1\ \Omega\ \right\} \end{aligned}$$

Result

a) $Z = 3\ \Omega$

b) $P_Z = \frac{3}{32}\ W$; $i_Z(t) = \frac{1}{4} \cdot sin(\omega t)\ A$

Problem 3.8

For the association of quadripoles shown in Figure 1(a), the three networks Q are identical and their internal structure is shown in figure 1(b). If the load impedance Z_L is equal to the characteristic impedance of the networks, obtain:

a) The image parameters of the quadripole Q.

b) The time-domain expression of voltage $V_{CD}(t)$.

c) The maximum number of quadripoles Q that we can connect in cascade, between the source (E_g, R_g) and the load Z_L in order to guarantee that the load receives a power $\geq 1\ mW$.

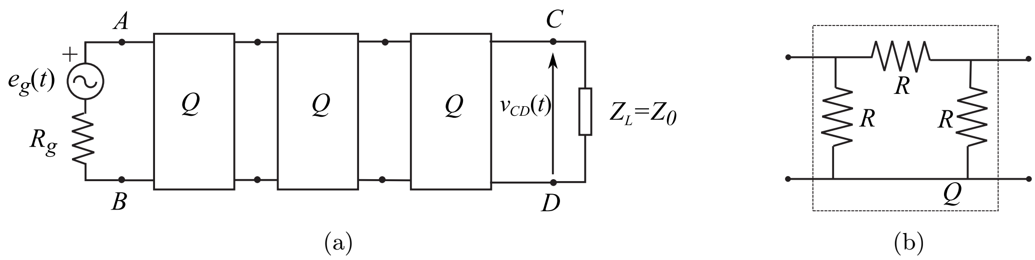

Figure 1

$$Data: \quad e_g(t) = 75\ \cos\,(\omega t)\ \text{V} \quad ; \quad R = 6\ \Omega \quad ; \quad R_g = 2Z_0$$

Result

a) $Z_0 = 2\sqrt{3}\ \Omega$; $\gamma = 1.3170$ Np

b) $V_{CD} = 0.4809$

c) $n = 4$

Problem 3.9

The quadripole shown in Figure 1(a), which is bilateral and symmetric, has the following parameters: $z_{11} = 8\ \Omega$ and $y_{11} = \frac{1}{2}\ \Omega^{-1}$.

a) Obtain the image parameters Z_0 and γ.

b) Determine the power absorbed by the load Z_L, using the circuit shown in Figure 1(b), where the quadripoles have the same internal structure of the two-port networks shown in Figure 1(a).

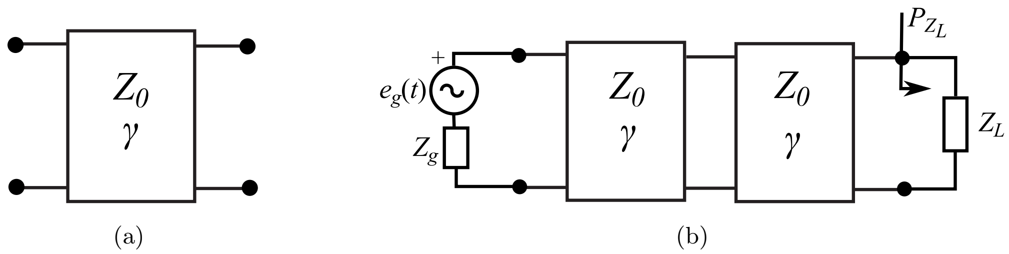

Figure 1

$$Data: \ \ e_g(t) = 4\sin(\omega t)\ \text{V}; \quad Z_g = Z_L = Z_0$$

Result

a) $Z_0 = 4\ \Omega$, $\gamma = \dfrac{\text{Ln}(3)}{2}\ Np$

b) $P_2 = \dfrac{1}{18}\ W$

Problem 3.10

For the circuit shown in Figure 1 all the quadripoles are identical and symmetric. The measurements shown in Figure 2 have been done with these quadripoles.

a) Compute the image parameters (Z_0 and γ) for the quadripole Q.

b) Determine the value of a so as to guarantee that the source ($e_g(t)$, Z_g) delivers the maximum power.

c) Obtain the power absorbed by the load Z_L and the time-domain expression of the current $i_L(t)$.

d) Determine the power transmission and insertion loss introduced by the set of quadripoles and the transformer.

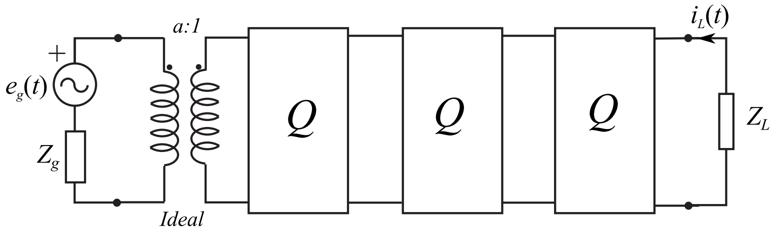

Figure 1

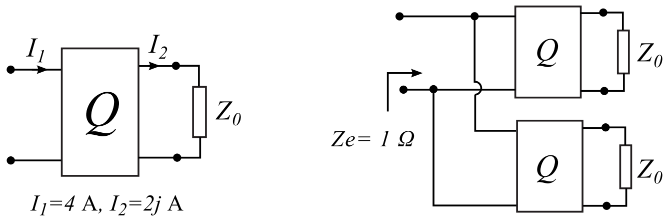

Figure 2

$$Data: \ \ e_g(t) = 16 \cdot \cos(\omega t)\ V. \ \ ; \ \ Z_g = 4Z_0 \ \ ; \ \ Z_L = Z_0.$$

Result

a) $\begin{cases} Z_0 = 2\ \Omega \\ \gamma = Ln(2) - j\frac{\pi}{2} \end{cases}$

b) $a = 2$

c) $P_{Z_L} = \frac{1}{16}\ W$; $i_L(t) = \frac{1}{4}\ cos\left(\omega t + \frac{3\pi}{2}\right)\ A$

d) $\alpha_T = 2.08\ Np$; $\alpha_I = 1.86\ Np$

Problem 3.11

For the circuit shown in Figure 1 determine the power of the source $i(t)$.

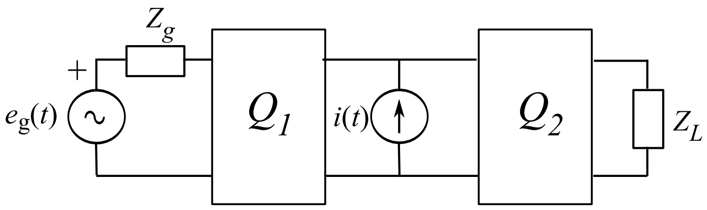

Figure 1

Data:

$e_g(t) = 8 \cdot sin(\omega t)\ V$; $i(t) = cos(\omega t)\ A$; $Z_g = Z_L = 3\ \Omega$

$Q_1 \equiv \begin{cases} Z_{0_1} = 3\ \Omega \\ Th(\gamma_1) = \frac{3}{5} \end{cases}$; $Q_2 \equiv \begin{cases} Z_{0_2} = 5\ \Omega \\ \gamma_2 = -j\frac{\pi}{3} \end{cases}$

Result

$$P_I = 0.8260 \text{ W}$$

Problem 3.12

In the circuit of Figure 1 we know that each quadripole Q is RLC. We also know that its transmission parameters are: $A = 3, \quad B = 8\ \Omega, \quad C = 1\ \Omega^{-1}, \quad D = 3.$

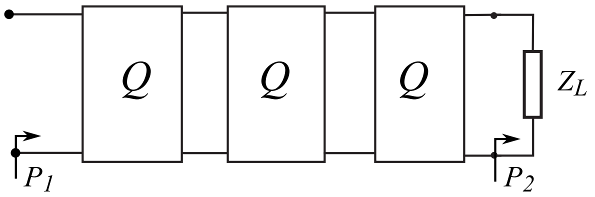

Figure 1

Data: $Z_L = 2\sqrt{2}\ \Omega$

a) Justify if quadripole Q is bilateral and symmetric. If so, obtain its image parameters (Z_0 and γ).

b) Find the ratio between the input power, P_1, and the output power, P_2, of the association given in Figure 1.

c) Compute the ratio between the power absorbed by the load before and after one of the quaripoles is removed from the association shown in Figure 1.

Result

a) Quadripole bilateral and symmetric. $\begin{cases} Z_0 = 2\sqrt{2}\ \Omega \\ \gamma = 1.763 \text{ Np} \end{cases}$

b) $\frac{P_1}{P_2} = 39261.5$

c) $\frac{P_2'}{P_2} = 33.99$

Problem 3.13

In the circuit shown in Figure 1, the three bilateral and symmetric quadripoles are characterized by their image parameters. They all have the same characteristic impedance Z_0. Moreover, we know that $Z_g = Z_L = Z_0$.

a) Compute the power absorbed by the load Z_L.

b) Compute the power of the voltage sources $e(t)$ and $e_g(t)$, and of the current source $i(t)$.

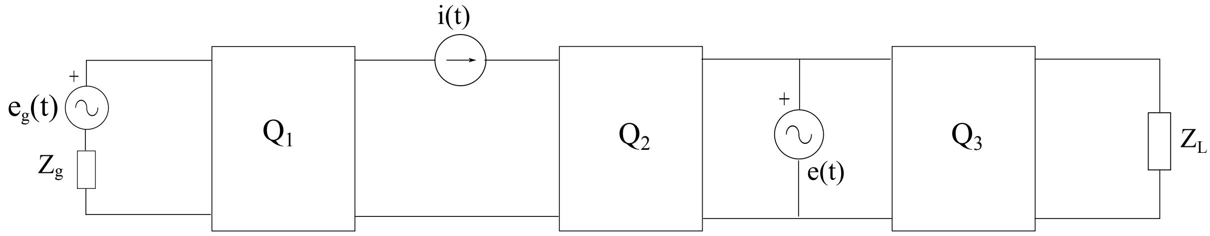

Figure 1

$$Data: \; e_g(t) = 30 \sin\left(\omega t + \tfrac{\pi}{2}\right) \; V; \;\; e(t) = 10 \sin(\omega t) \; V; \;\; i(t) = 5 \sin(\omega t) \; A;$$

$$Q_1 \begin{cases} Z_0 = 5 \; \Omega \\ \gamma_1 = j\frac{\pi}{2} \end{cases} \quad Q_2 \begin{cases} Z_0 = 5 \; \Omega \\ \gamma_2 = j\pi \end{cases} \quad Q_3 \begin{cases} Z_0 = 5 \; \Omega \\ \gamma_3 = ln(2) \text{ Np} \end{cases}$$

Result

a) $P_{Z_L} = \frac{5}{2} \; W$

b) $P_{E_g} = 15 \; W$; $P_E = 35 \; W$; $P_I = -\frac{75}{2} \; W$

Problem 3.14

For the association of quadripoles shown in Figure 1, it is known that the two-port network Q_2 is bilateral, and that its output impedance with an open-circuit input is $Z_{2A} = \dfrac{1}{2} \; \Omega$. Moreover, the measurements shown in Figure 2 have been done.

a) Determine a family of parameters to characterize the quadripole Q_2.

b) Find the value for resistor R in order to guarantee that the source delivers the maximum power.

c) Determine the transmission loss of the network between the real source (E_g, Z_g) and the load (Z_L).

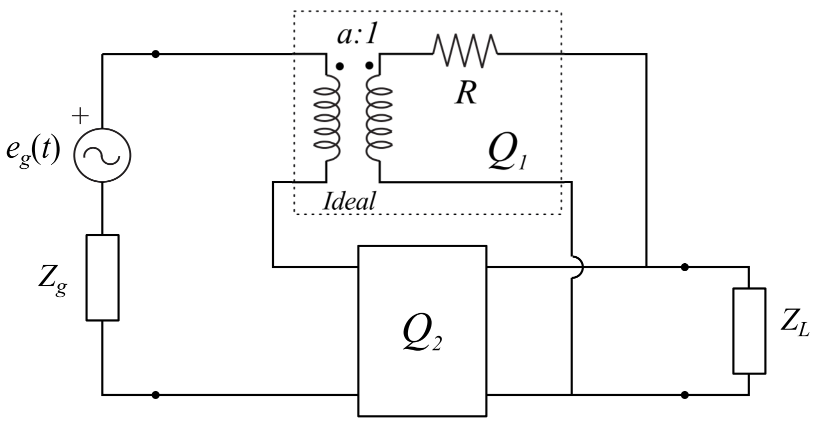

Figure 1

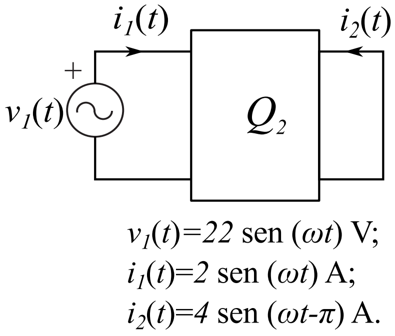

Figure 2

Data: $Z_g = 35\ \Omega$; $Z_L = \frac{1}{2}\ \Omega$; $a = 2$.

Result

a) $h_{Q_2} = \begin{pmatrix} 11\ \Omega & 2 \\ -2 & 2\ \Omega^{-1} \end{pmatrix}$

b) $R = 5\ \Omega$.

c) $\alpha_T = 1.4311$ Np $= 12.43$ dB.

Problem 3.15

Given the association of quaripoles in Figure 1, determine:

a) The image parameters of quadripoles Q_1 and Q_2.

b) The insertion loss produced by quaripole Q_2 and the transmission loss of quadripole Q_1.

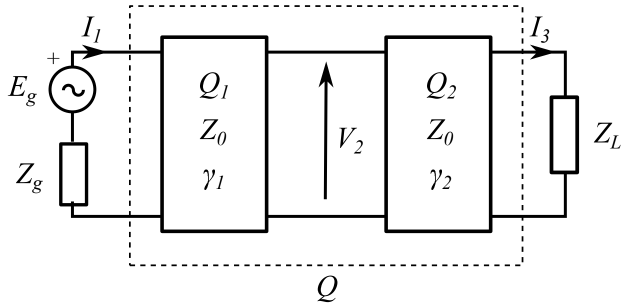

Figure 1

$$Data: \quad E_g = 100\text{ V} \quad ; \quad Z_g = (6-2j)\ \Omega \quad ; \quad Z_L = Z_0 \quad ; \quad I_1 = (4+3j)\text{ A} \quad ; \quad V_2 = (7-j)\text{ V}$$
$$I_3 = -\frac{4+3j}{10}\text{ A}$$

Result

a) $Z_0 = (10 - 10j)\ \Omega$, $\gamma_1 = \text{Ln}(10)$ Np, $\gamma_2 = j\pi$ rad.

b) $\alpha_{I_2} = \alpha_2 = 0$ Np, $\alpha_{T_1} = \alpha_1 = \text{Ln}(10)$ Np

Problem 3.16

Quaripoles Q_A and Q_B have been connected as it is shown in Figure 1. Of quaripole Q_A we know the value of $Z_{0A} = \frac{1-j}{3}\Omega$. Of quaripole Q_B we know that it is bilateral and symmetric, and we also know its transmission parameters A and B.

a) Compute the image parameters of quadripole Q_B.

b) Determine the time-domain expression of the current $i_L(t)$, if the load Z_L coincides with the characteristic impedance of the quadripole Q_B.

c) Compute the transmission and insertion loss of the set of two quadripoles Q_B connected in cascade.

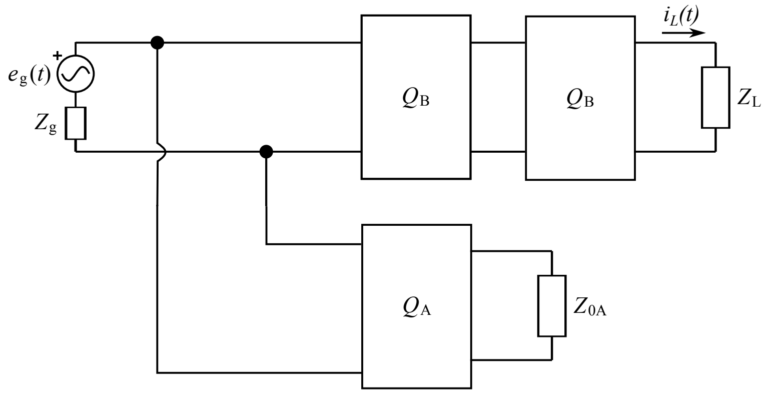

Figure 1

Data:

$$e_g(t) = 2\cos(\omega t)\ V \quad ; \quad Z_g = \frac{1+j}{4}$$

$$Q_B \begin{cases} A = \dfrac{1}{\sqrt{2}} \\ B = \dfrac{1+j}{\sqrt{2}}\Omega \end{cases}$$

Result

a) $Z_{0B} = (1-j)\ \Omega$ y $\gamma_B = j\dfrac{\pi}{4}$.

b) $i_L(t) = \cos\left(\omega t - \frac{\pi}{2}\right)$ A.

c) $\alpha_T = \alpha_I = 2\alpha_B = 0$ Np .

Problem 3.17

In the circuit of Figure 1, quadripole Q_A is bilateral and symmetric, with transmission parameters $B = 4\ \Omega$ and $D = 3$, and the source $(e_g(t), R_g)$ is delivering the maximum power. With these conditions, we know that $i_x(t) = \cos(\omega t - \pi/4)\ A$. Determine:

a) The valur of the characteristic impedance of quadripole Q_B, Z_{0B}.

b) The value of γ_B for quadripole Q_B.

c) The time-domain expresion of the voltage at the output of quadripole Q_A, $v(t)$.

d) Transmisison and insertion loss for the quadripole Q_B. Justify your answer.

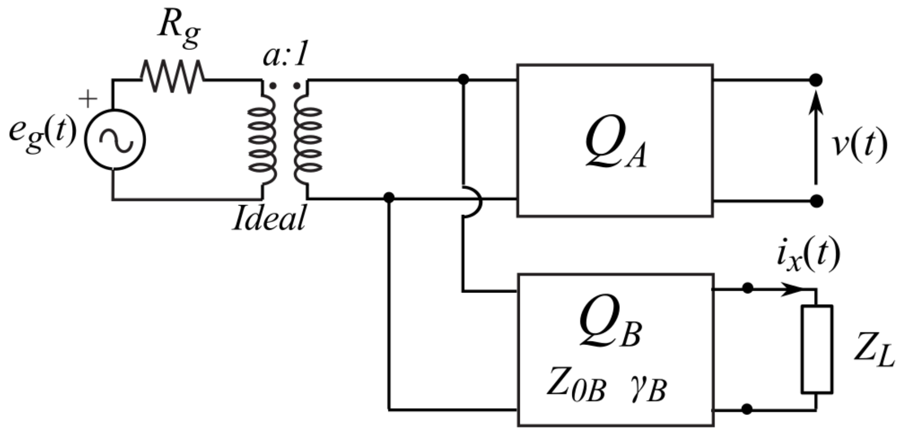

Figure 1

Data: $e_g(t) = 12\cos(\omega t)$ V ; $R_g = 3\ \Omega$; $a = 2$; $Z_L = Z_{0B}$

Result

1. $Z_{0B} = \dfrac{3}{2}\ \Omega$
2. $\gamma_B = \ln(2) + j\dfrac{\pi}{4}$
3. $v(t) = \cos(\omega t)$ V
4. $\alpha_T = \alpha_I = \Re e\{\gamma_B\} = \alpha_B = \ln(2)$ Np