PROBLEMS OF CHAPTER 4: INTRODUCTION TO PASSIVE FILTERS.

April 4, 2017

Problem 4.1

For the circuit shown in Figure 1(a) we want to design a filter with the zero-pole diagram shown in Figure 1 (b).

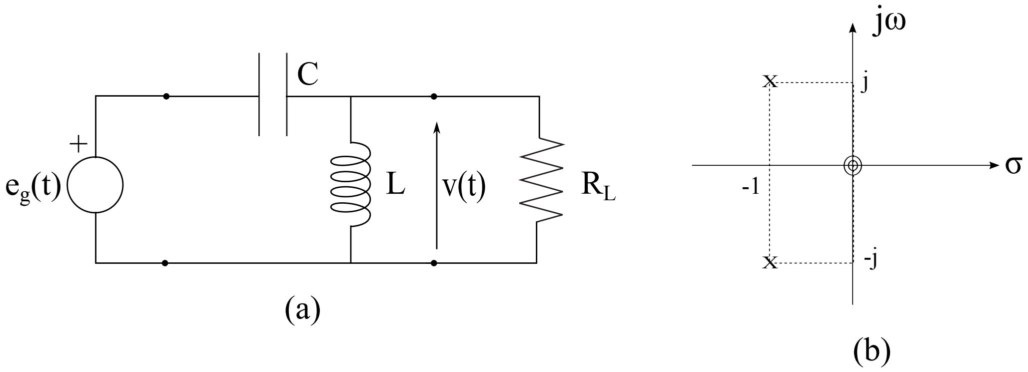


Figure 1

a) Justify the type of filter proposed.

b) Obtain the literal expression of the transfer function of the circuit, which is defined as $H(s) = \frac{V(s)}{E_g(s)}$, as a function of the components of the circuit.

c) Obtain the values for L and C in order to build a circuit that satisfies the zero-pole diagram given. Consider $R_L = 1\ \Omega$.

Result

a) High-pass filter.

b) $H(s) = \dfrac{s^2}{s^2 + \dfrac{1}{CR_L}s + \dfrac{1}{LC}}$

c) $K = 1$, $C = \frac{1}{2}\ F$, $L = 1\ H$

Problem 4.2

Given the circuit of Figure 1:

a) Obtain the literal expression of the transfer function of the circuit, as a function of the components of the circuit, defined as $H(s) = \dfrac{V_2(s)}{E_g(s)}$.

b) Plot the zero-pole diagram, and approximately represent the modulus of the frequency response of the filter. Justify the type of filter obtained.

c) Compute $v_2(t)$ considering that $e_g(t) = 3 \cdot \text{sen}(4t)\ V$.

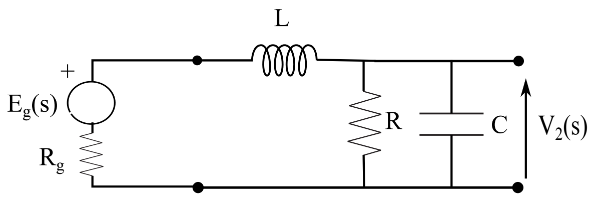


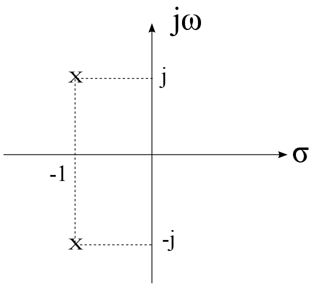
Figure 1

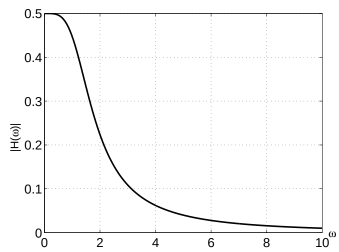
Data: $R = R_g = 1\,\Omega \quad ; \quad L = 1H \quad ; \quad C = 1F.$

Result

a) $$H(s) = \frac{R}{R_g + R} \cdot \frac{\dfrac{R_g + R}{LCR}}{s^2 + \left(\dfrac{R_g}{L} + \dfrac{1}{RC}\right) s + \dfrac{R_g + R}{LCR}}$$

b) Low-pass filter.





c) $v_2(t) = 0.186\ \text{sen}(4t - 2.6224)$ V

Problem 4.3

For the circuit in Figure 1(a), it transfer function $H(s) = \frac{V_{R_L}(s)}{E_g(s)}$ has the zero-pole diagram shown in Figure 1(b). Moreover, we know that $H(s)|_{s=0} = \frac{1}{2}$ and that $R_L = 1\,\Omega$.

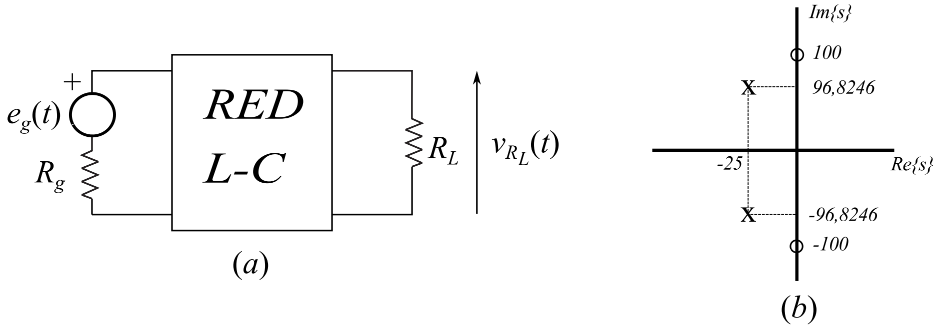


Figure 1

a) Justify the type of filter provided.

b) Computer the parameters of the filter:

- H_0 and cur frequency.
- Central frequency and bandwidth.

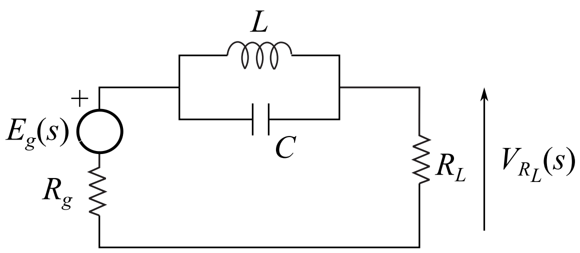
c) Compute the value of R_g and a possible $L-C$ network specifying the value of all the components..

Result

a) Bad-stop filter.

b) $H_0 = \dfrac{1}{2}$, $\omega_0 = 100$ rad/s, $B = 50\ rad/s$, $Q = \frac{\omega_0}{B} = 2$, $\omega_{C_i} = 78.078$ rad/s, $\omega_{C_s} = 128.078$ rad/s

c) $R_g = 1\ \Omega$, $C = 10$ mF, $L = 10$ mH



Problem 4.4

For the circuit in Figure 1 we can consider that the initial conditions are zero. Determine:

a) The transfer function $H(s) = \frac{V_S(s)}{E_g(s)}$.

b) Justify the type of filter provided, as well as its main characteristics (H_0, cut frequency, ...).

c) Draw the zero-pole plot and represent the modulus of the frequency response of the filter..

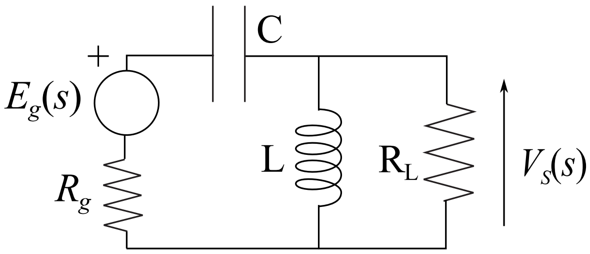


Figure 1

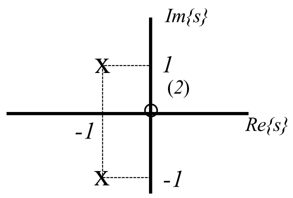
Data: $R_g = R_L = 1\ \Omega$, $L = \dfrac{1}{2}$ H, $C = \dfrac{1}{2}$ F

Result

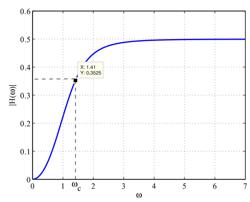
a) $H(s) = \dfrac{1}{2} \cdot \dfrac{s^2}{s^2 + 2s + 2}$

b) High-pass filter, $H_0 = \frac{1}{2}$, $\omega_c = \sqrt{2}$ rad/s.

c)



(a)



(b)

Problem 4.5

Given the circuit of Figure 1:

a) Obtain the literal expression of the transfer function $H(s) = \dfrac{V_S(s)}{E_g(s)}$ as a function of R_L, C and L.

b) Determine the value of C in order to get the minimum value of the modulus of the frequency response for $\omega_1 = 100$ rad/s.

c) Draw the zero-plot diagram and plot the modulus of the frequency response. Justify the type of filter obtained.

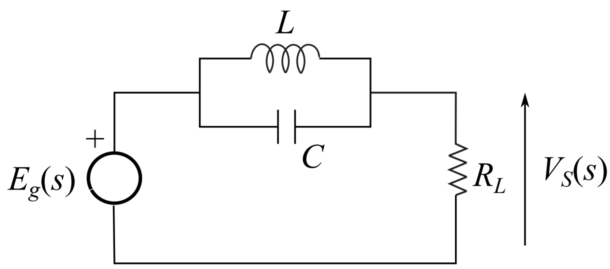


Figure 1

Data: $R_L = 2\ \Omega; \quad L = 10$ mH

Result

a) $H(s) = \dfrac{s^2 + \dfrac{1}{LC}}{s^2 + \dfrac{1}{R_L C}s + \dfrac{1}{LC}}$

b) $C = 10$ mF

c) Stop-band filter.

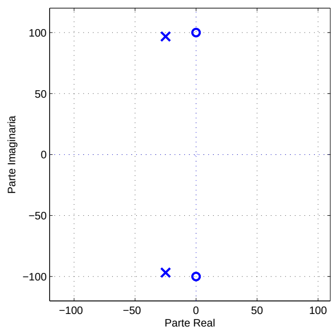


Figure 2

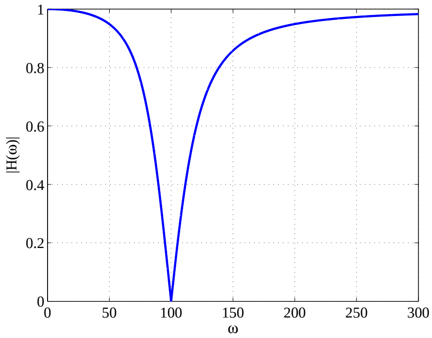


Figure 3

Problem 4.6

For the circuit in Figure 1:

a) Obtain the literal expression of the transfer function $H(s) = \dfrac{V_S(s)}{E_g(s)}$. Justify the type of filter provided.

b) Obtain the value of R_L in order to get a bandwidth of $B = 1$ Krad/s.

c) Plot the zero-pole diagram, and draw the modulus of the frequency response.

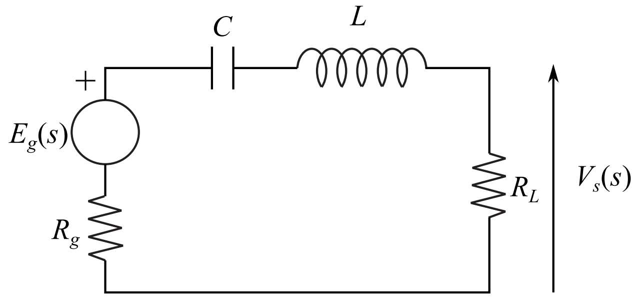


Figure 1

Data: $R_g = 50\ \Omega$; $L = 100$ mH; $C = 100$ nF

Result

a) Band-pass filter. $H(s) = \dfrac{\dfrac{R_L}{L}s}{s^2 + \dfrac{R_g + R_L}{L}s + \dfrac{1}{LC}}$

b) $R_L = 50\ \Omega$

c) Zero-pole plot $|H(\omega)|$:

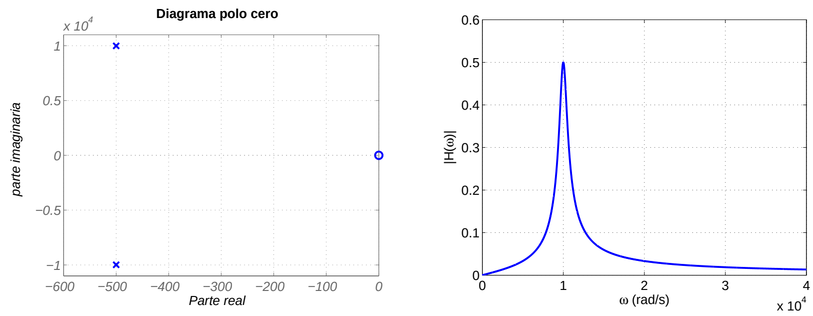


Figure 2

Problem 4.7

Given the circuit of Figure 1,

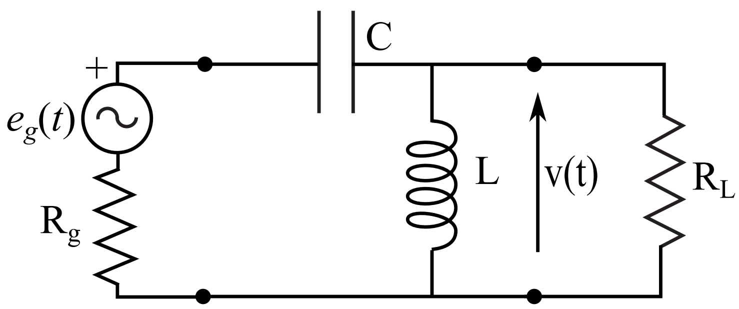


Figure 1

a) Obtain the literal expression of the transfer function $H(s) = \frac{V(s)}{E_g(s)}$. Compare this transfer function with the general form of transfer functions for second-order filters $H(s) = \dfrac{a_2 s^2 + a_1 s + a_0}{s^2 + b_1 s + b_0}$ and determine a_2, a_1, a_0, b_1, b_0.

b) Justify the type of filter provided considering the modulus of its frequency response.

c) If $e_g(t) = 8 + 3 \cdot \cos(20t)V$, determine $v(t)$.

Data:

$$R_g = R_L = 1\,\Omega; \qquad C = 10mF; \qquad L = 10mH$$

Result

a) $$H(s) = \frac{V(s)}{E_g(s)} = \frac{R_L}{R_g + R_L} \cdot \frac{s^2}{s^2 + \dfrac{R_L}{R_g + R_L} \cdot \left(\dfrac{R_g}{L} + \dfrac{1}{R_L C}\right) s + \dfrac{R_L}{R_g + R_L} \cdot \dfrac{1}{LC}}$$

b) High-pass filter.

c) $v(t) = 0.12 \cdot \cos(20t + 2.7315)$ V

Problem 4.8

Given the circuit of Figure 1,

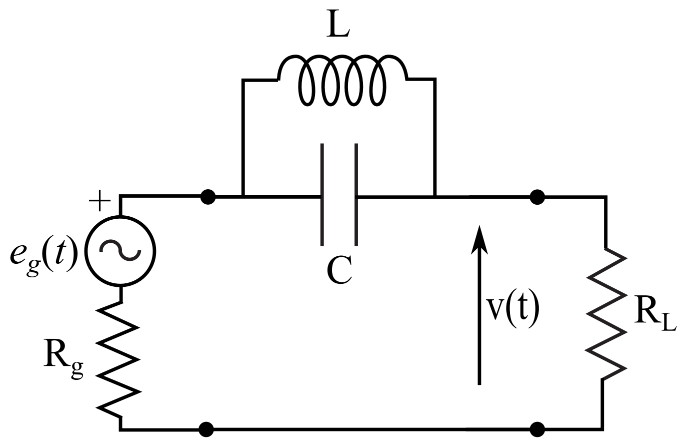


Figure 1

a) Obtain the literal expression of the transfer function $H(s) = \frac{V(s)}{E_g(s)}$. Compare this transfer function with the general form of transfer functions for second-order filters $H(s) = \dfrac{a_2 s^2 + a_1 s + a_0}{s^2 + b_1 s + b_0}$ and determine a_2, a_1, a_0, b_1, b_0.

b) Justify the type of filter provided considering the modulus of its frequency response.

c) Compute the value of C in order to eliminate the frequency $\omega = 100\,rad/s$.

d) If $e_g(t) = 8 \cdot \sin(100t) + 3 \cdot \cos(20t)V$, determine $v(t)$.

Data:

$$R_g = R_L = 2\,\Omega; \qquad L = 10mH$$

Result

a)

$$H(s) = \frac{V(s)}{E_g(s)} = \frac{\dfrac{R_L}{R_g + R_L} \cdot \left(s^2 + \dfrac{1}{LC}\right)}{s^2 + \dfrac{1}{(R_g + R_L)\,C} s + \dfrac{1}{LC}}$$

$$\begin{cases} a_2 = \dfrac{R_L}{R_g + R_L} \\ a_1 = 0 \\ a_0 = \dfrac{R_L}{R_g + R_L} \cdot \dfrac{1}{LC} \\ b_1 = \dfrac{1}{(R_g + R_L)\,C} \\ b_0 = \dfrac{1}{LC} \end{cases}$$

b) Band-stop filter.

c) $C = 10$ mF.

d) $v(t) = 1.498 \cdot \cos\left(20t - 0.052\}\right)$ V

Problem 4.9

For the circuit in Figure 1, determine:

a) The literal expression of the transfer function $H(s) = \frac{V_s(s)}{E_g(s)}$.

b) Values of L and C to get a zero-pole plot of $H(s)$ like the one shown in Figure 2, considering a bandwidth of $B = 10^3$ rad/s.

c) $v_s(t)$ when the signal provided by the source is $e_g(t) = 4 + 3 \cdot \text{sen}\left(10^3 t + \dfrac{\pi}{4}\right)$

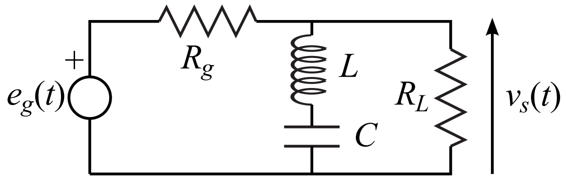


Figure 1

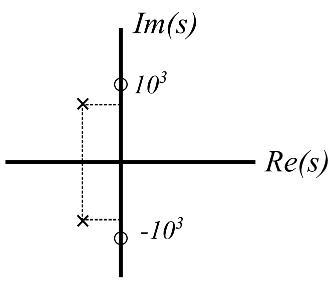


Figure 2

Data: $R_g = R_L = 1\,\Omega$

Result

a) $H(s) = \dfrac{V(s)}{E_g(s)} = \dfrac{R_L}{R_L + R_g} \cdot \dfrac{s^2 + \dfrac{1}{LC}}{s^2 + \dfrac{R_g R_L}{(R_L + R_g)L} \cdot s + \dfrac{1}{LC}}$

b) $L = \dfrac{1}{2}$ mH, $C = 2$ mF.

c) $v_s(t) = 2$ V

Problem 4.10

For the circuit in Figure 1:

a) Obtain the literal expression of the transfer function $H(s) = \frac{V_s(s)}{E_g(s)}$ and its zero-pole plot.

b) Represent the modulus of the frequency response of the filter, indicating the most representative values (maximum value and its frequency, bandwidth). Justify the type of filter provided.

c) Obtain the signal $v_s(t)$ when $e_g(t) = 3 \cdot \cos(10^4\ t) + 3 \cdot \text{sen}(2 \cdot 10^4\ t)\, V$.

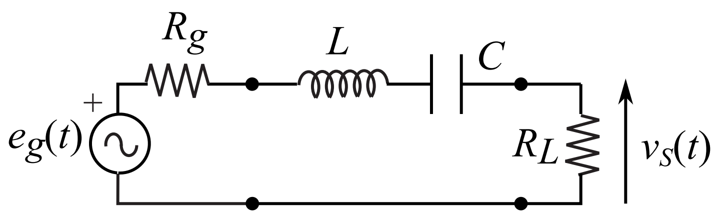


Figure 1

Data: $R_g = 0,5\ \Omega$; $L = 1$ mH ; $C = 10\ \mu$F ; $R_L = 1,5\ \Omega$

Result

a) $H(s) = \dfrac{R_L}{R_g + R_L} \cdot \dfrac{\dfrac{R_g + R_L}{L} s}{s^2 + \dfrac{R_g + R_L}{L} s + \dfrac{1}{LC}}$

$$s_{p_{1,2}} = \left(-1 \pm j\sqrt{99}\right) \cdot 10^3 \quad ; \quad s_{C_1} = 0 \quad ; \quad s_{C_2} \to \infty$$

b) Pass-badn filter. $\omega_0 = 10^4$rad/s, $H(\omega = 10^4) = \dfrac{3}{4}$, $B = 2 \cdot 10^3$ rad/s.

c) $v_s(t) = \frac{9}{4} \cdot \cos\left(10^4 t\right) + \frac{9}{2\sqrt{229}} \cdot \text{sen}\left(2 \cdot 10^4 t - 1.4382\right)$ V