Transient circuit's analysis using Laplace transforms.

The study of the circuit's transient behavior will be done in the Laplace transformed domain to avoid the more tedious resolution of the differential equations. However, it is important to have understood well the concepts we have seen by solving the circuits in the time domain, i.e., to know which is the behavior of the solution we are going to obtain by solving a second order circuit. Remember that we can predict the kind of solution if we know the roots of the *characteristic equation*: if they are real and different → overdamped behavior; equal → critically damped behavior; complex conjugated → underdamped behavior; pure imaginary → undamped behavior. The real part is always negative and is the exponent of the exponential decrease of the solution, and if the imaginary part is non zero the solution will oscillate with the time (underdamped or undamped).

To obtain the transient behavior of a circuit:

1. Obtain the initial conditions: voltages at the capacitors and current trough the coils just before the switching takes place at $t=t_0$. Sometimes the initial conditions are easily obtained when the source are continuous, but when the source are sinusoidal you may have to do a first resolution using fasors and particularize the solution for $t=t_0$ to obtain the initial conditions.

2. If the switching instant is different than zero ($t_0 \neq 0$), define a new displaced time $t' = t - t_0$, so that the integral intervals of the Laplace transform integral are defined between $t'=0$ and infinite.

3. After the switching takes place, "translate" the circuit into the Laplace domain (with variable s), remember:

   $g(t)$+switcher → $G(s)$,
   $L \to sL \,//\, i_L(t_0)/s = sL + i_L(t_0)L$.
   $C \to 1/(sC) + v_C(t_0)/s$.

4. Perform the circuit analysis in the Laplace domain (using the same Kirchoff's laws and circuit resolution rules used in the time domain) until you obtain an expression of the form (e.g. for a second order circuit):

   $$I(s)=\frac{f(s)}{s^2+as+b} ,$$

   being $I(s)=\mathcal{L}\{i(t)\}$, ($\mathcal{L}$: Laplace transform operator).

   Note: we will use capital letters ($V(s)$, $I(s)$) for variables in the Laplace domain and small letters ($v(t)$, $i(t)$) for the variables in the time domain.

5. Analyze the characteristic equation $s^2+\mathrm{a}s+\mathrm{b}=0$. Depending of the solution of the roots (s_1, s_2) you will know the kind of solutions you will obtain for $i(t)$. This will also help you to obtain the inverse Laplace transform using the *Laplace Transform Table* (LTT). For example:

   If $s_1 \neq s_2$ and real, use $s^2+\mathrm{a}s+\mathrm{b}=(s-s_1)(s-s_2)$, (nº 12 to 14 in the LTT)
   If $s_1=s_2$ use $s^2+\mathrm{a}s+\mathrm{b}=(s-s_1)^2$, (nº 8 in the LTT)
   If $s_1=s_2^*$ are complex, being $s=p+\mathrm{j}q$, use $s^2+\mathrm{a}s+\mathrm{b}=(s-s_1)(s-s_2)=(s-p)^2+q^2$, (nº 24 in the LTT).

6. Perform the inverse Laplace transform using the LTT: $i(t')=\mathcal{L}^{-1}\{I(s)\}$. If you have done a time-shifting (step 2), replace $t'=t-t_0$.

7. Check if the solution you have obtained fulfills the initial conditions.