U3. Power transfer theorems and image parameters

Circuit Analysis, GIT

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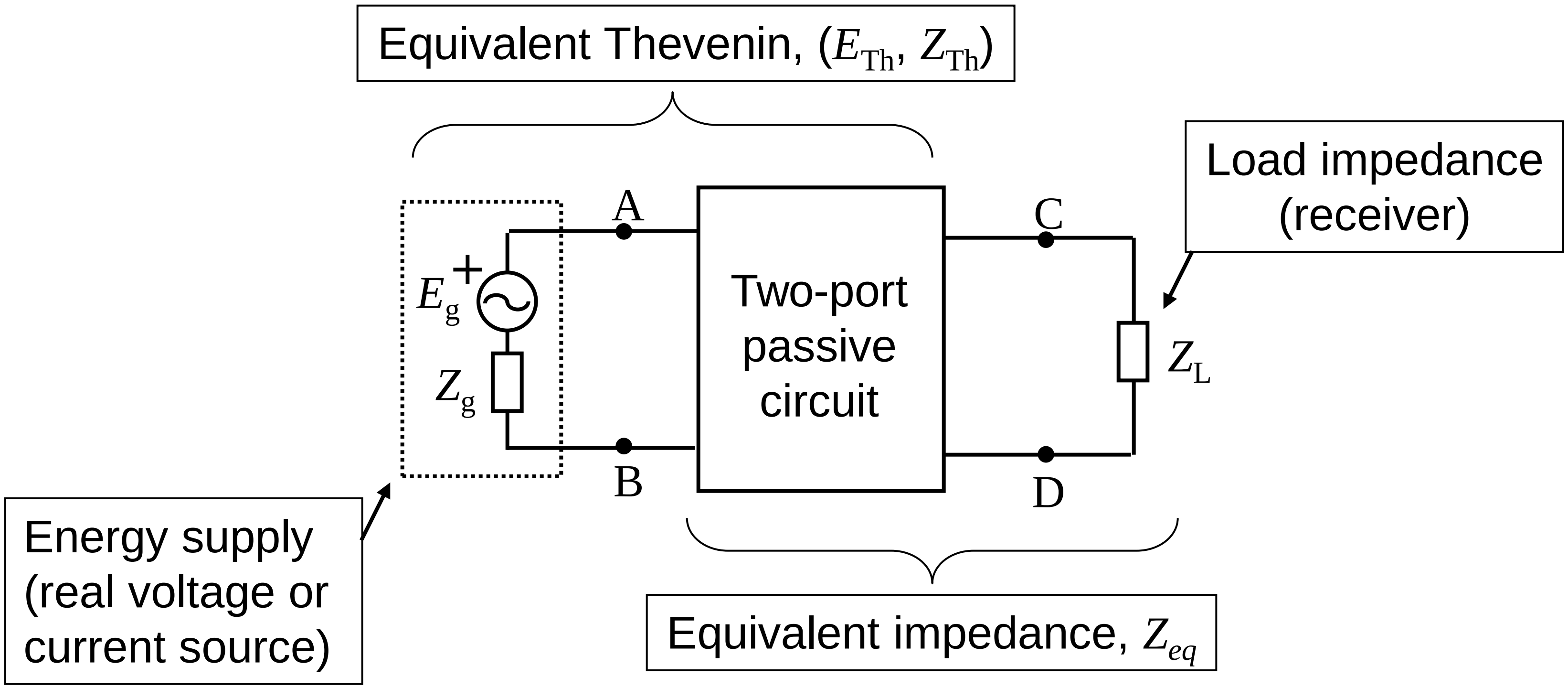
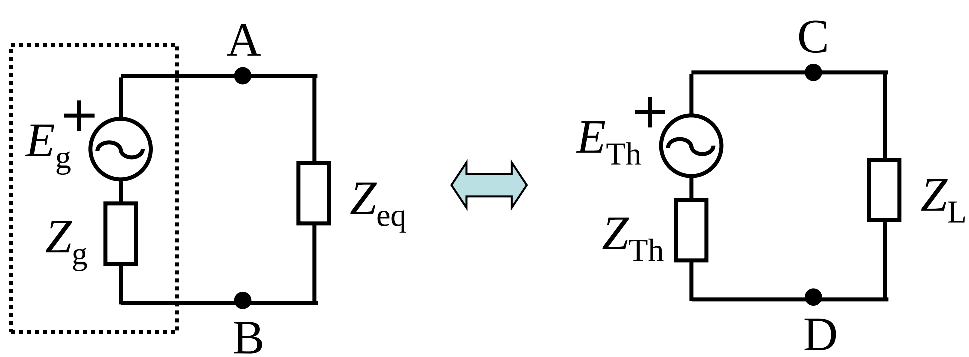
Index

- Introduction: Typical circuit configuration
- Recall
  - Equivalent impedance
  - Equivalent Thevenin and Norton
- Maximum power supply. Impedance Maching
- Power transfer. Everitt Theorem.
- Transmission Units
- Image Parameters
- Cascaded association using image param.

Typical circuit configuration

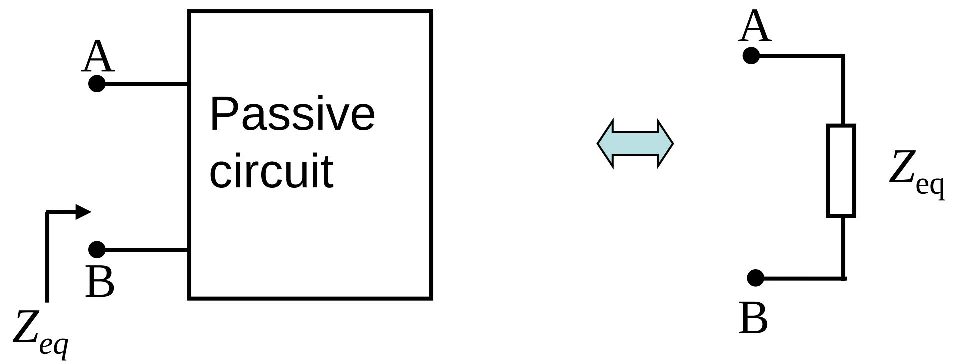


Equivalent circuits:



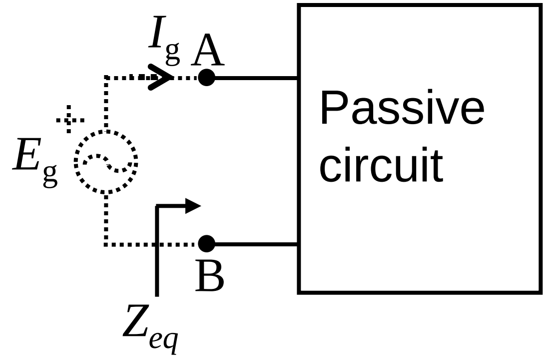
Recall: Equivalent Impedance

- Any passive circuit (i.e. without independent sources) between two terminals (A,B) can be simplified with a equivalent impedance, Z_{eq}:



The equivalent impedance can be obtained by:

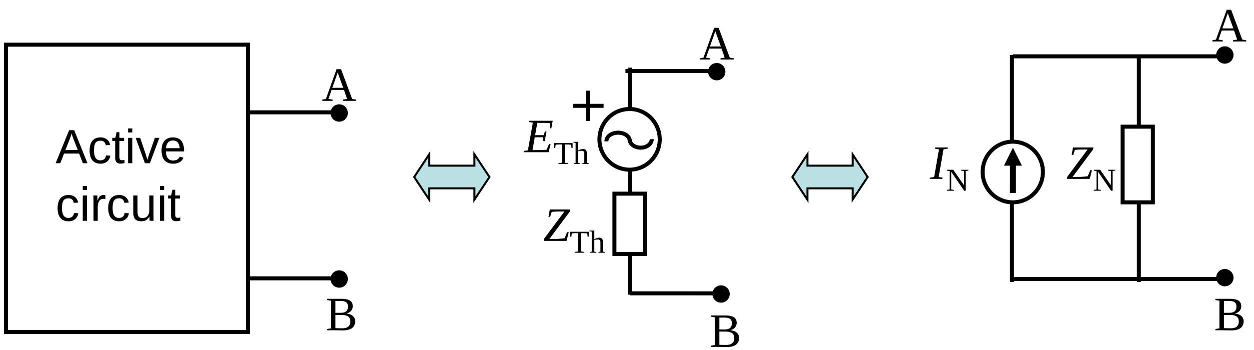
- Parallel and/or serial association of the impedances within the passive circuit.
- Applying a test generator E_g:



$$Z_{eq} = \frac{E_g}{I_g}$$

Recall: Equivalent Thevenin & Norton

- Any linear circuit with independent sources (i.e. active circuit) between two terminals (A,B) can be simplified by a real voltage source (Equiv.Thevenin) or real current source (Equiv. Norton):



Equivalent Thevenin

Equivalent Norton

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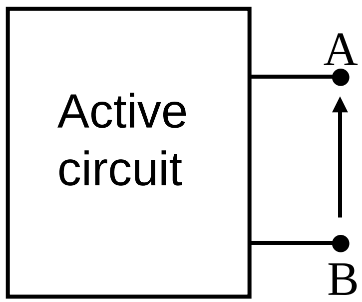
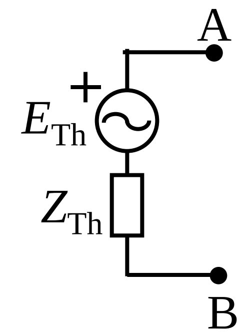
Recall: Equivalent Thevenin

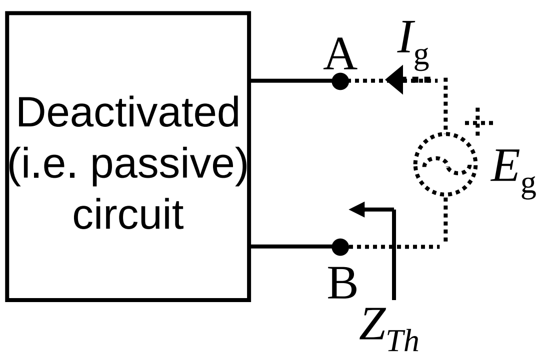
- E_{Th} is the voltage between A and B in open circuit:



$$V_{AB} = V_A - V_B = E_{Th} \text{ in o.c.}$$



- Z_{th} is the equivalent impedance seen trough A-B

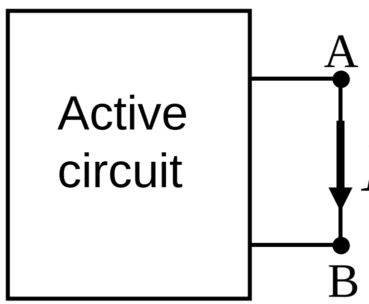


$$Z_{Th} = \frac{E_g}{I_g}, \text{ by annulling}^{(*)} \text{ the independend sources in the circuit}$$

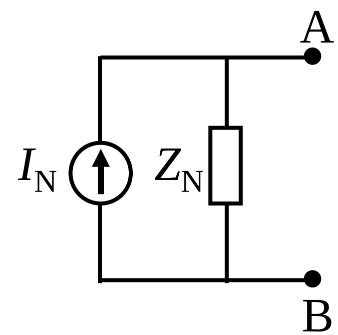
(*) see slide 4

Recall: Equivalent Norton

- I_N is the current between A and B in short circuit:

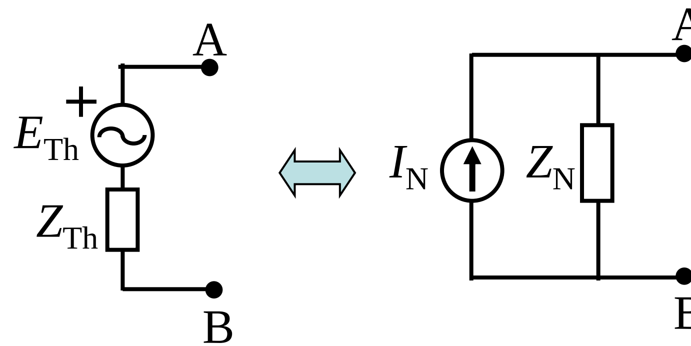


$$I_{AB} = I_N \text{ in s.c.}$$



- Z_N (as Z_{Th}) is the equivalent impedance seen trough A-B
- Equivalencies between the real Thevenin and Norton sources:

$$\begin{cases} E_{Th} = Z_N I_N \\ Z_{Th} = Z_N \end{cases}$$



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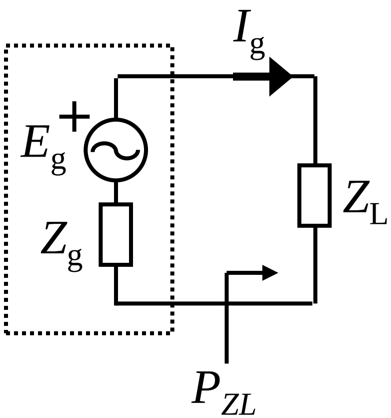
Maximum power supply

- When does a real generator (E_g, Z_g) supply the maximum power to a load impedance Z_L?



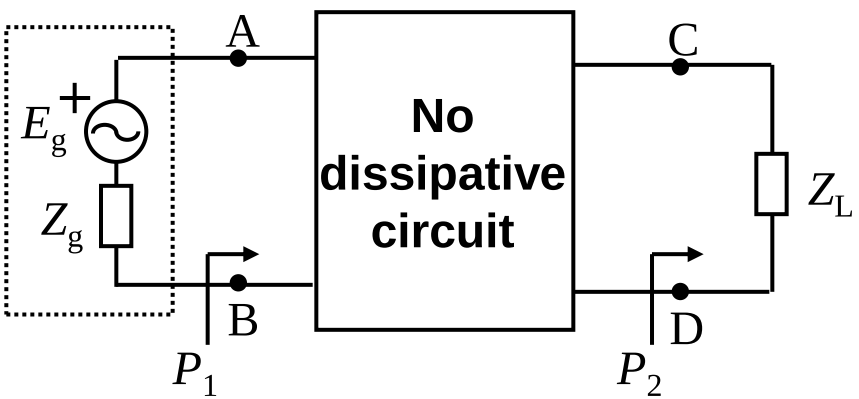
$$\begin{cases} Z_g = a + \mathrm{j}b & \text{fixed} \\ Z_L = x + \mathrm{j}y & \text{variable} \end{cases}$$

$$P_{ZL} = \frac{1}{2}\left|I_g\right|^2 \mathrm{Re}[Z_L] = \frac{\left|E_g\right|^2}{2}\frac{x}{\left|a + x + \mathrm{j}(b + y)\right|^2}$$

$$P_{ZL} = P_{ZL}\big|_{\max} \Leftrightarrow \begin{cases} x = a \\ y = -b \end{cases} \Leftrightarrow Z_L = Z_g^* \Leftrightarrow \textbf{Impedance matching} \text{ and } P_{ZL}\big|_{\max} = \frac{\left|E_g\right|^2}{8a}$$

Everitt theorem

- When there is maximum power supply at the input gate (P_1) of a **non-dissipative circuit** (i.e. made of *L* and *C* only) then there is also maximum power supply at the output (P_2) of the circuit and vice versa:



$$P_1 = P_1\big|_{\max} \Leftrightarrow P_2 = P_2\big|_{\max}$$

$$\text{Impedance matching at A - B} \Leftrightarrow \text{Impedance matching at C - D}$$

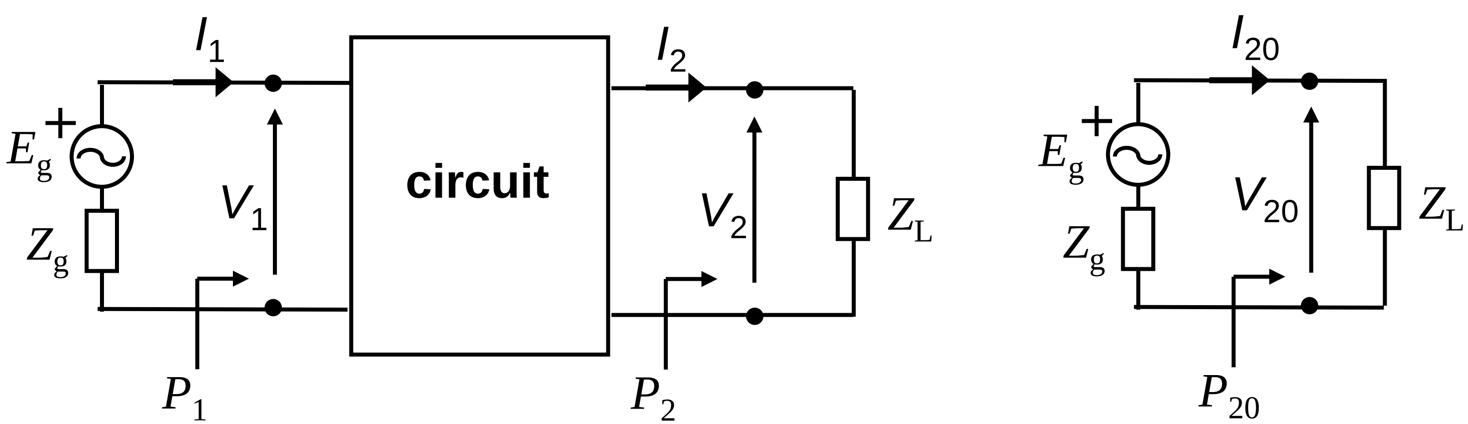
Transmission units

- For measuring the signal transmission trough a circuit the following magnitudes are usually used:
  - Transmission loss:

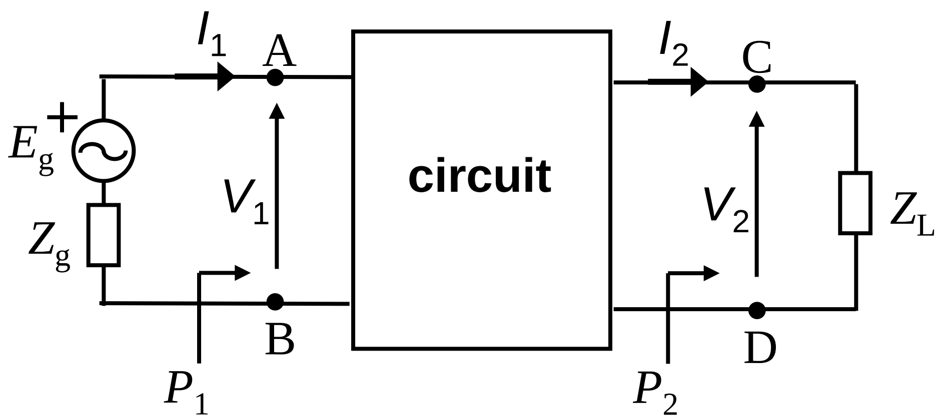
$$\frac{\text{circuits imput signal}}{\text{circuits output signal}}$$

  - Insertion loss:

$$\frac{\text{supplied signal without circuit}}{\text{circuits output signal}}$$

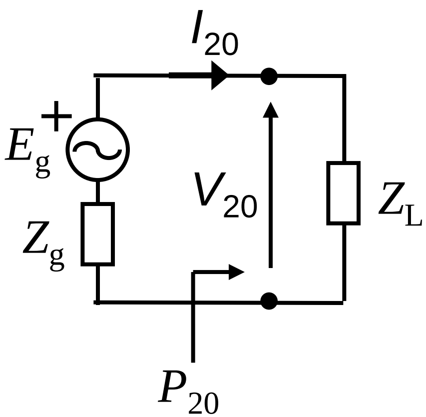


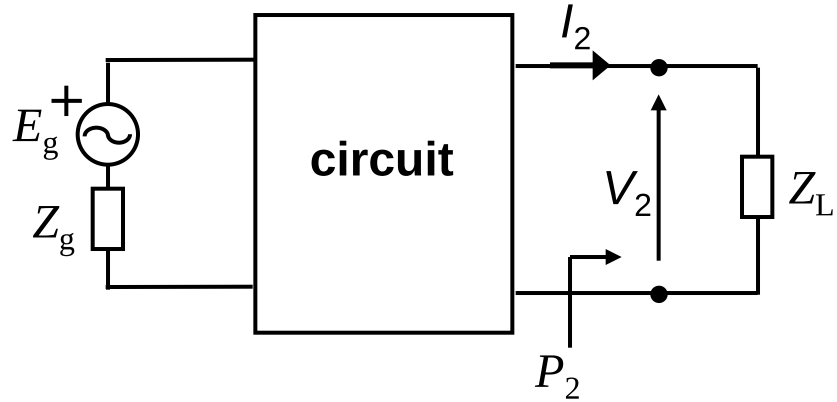
Transmission loss



	Corriente	Tensión	Potencia
U. naturales	$l_T = \frac{\vert I_1 \vert}{\vert I_2 \vert}$	$l_T = \frac{\vert V_1 \vert}{\vert V_2 \vert}$	$l_T = \frac{P_1}{P_2}$
Decibelios	$L_T = 20 \log \frac{\vert I_1 \vert}{\vert I_2 \vert} [\mathrm{dB}]$	$L_T = 20 \log \frac{\vert V_1 \vert}{\vert V_2 \vert} [\mathrm{dB}]$	$L_T = 10 \log \frac{P_1}{P_2} [\mathrm{dB}]$
Neperios	$L_T = \ln \frac{\vert I_1 \vert}{\vert I_2 \vert} [\mathrm{Np}]$	$L_T = \ln \frac{\vert V_1 \vert}{\vert V_2 \vert} [\mathrm{Np}]$	$L_T = \frac{1}{2} \ln \frac{P_1}{P_2} [\mathrm{Np}]$

Insertion loss





	Corriente	Tensión	Potencia
U. naturales	$l_I = \dfrac{\lvert I_{20}\rvert}{\lvert I_2\rvert}$	$l_I = \dfrac{\lvert V_{20}\rvert}{\lvert V_2\rvert}$	$l_I = \dfrac{P_{20}}{P_2}$
Decibelios	$L_I = 20\log\dfrac{\lvert I_{20}\rvert}{\lvert I_2\rvert}[\mathrm{dB}]$	$L_I = 20\log\dfrac{\lvert V_{20}\rvert}{\lvert V_2\rvert}[\mathrm{dB}]$	$L_I = 10\log\dfrac{P_{20}}{P_2}[\mathrm{dB}]$
Neperios	$L_I = \ln\dfrac{\lvert I_{20}\rvert}{\lvert I_2\rvert}[\mathrm{Np}]$	$L_I = \ln\dfrac{\lvert V_{20}\rvert}{\lvert V_2\rvert}[\mathrm{Np}]$	$L_I = \dfrac{1}{2}\ln\dfrac{P_{20}}{P_2}[\mathrm{Np}]$

Are the different insertion loss the same if in dB and Np?

Transmission units

- Transmission gain:

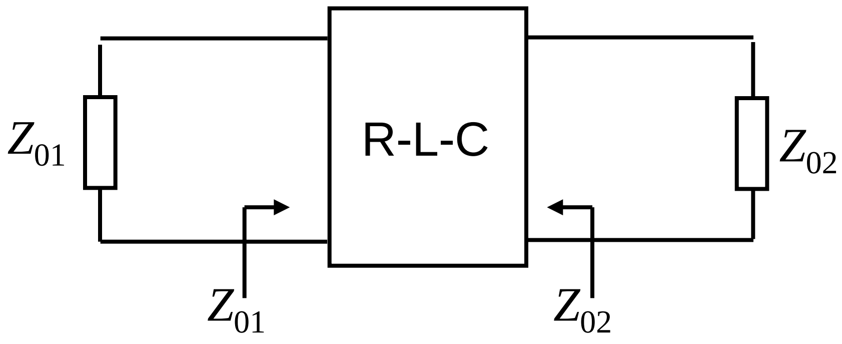
	Corriente	Tensión	Potencia
U. naturales (g=1/l)	$g_T = \frac{\lvert I_2 \rvert}{\lvert I_1 \rvert}$	$g_T = \frac{\lvert V_2 \rvert}{\lvert V_1 \rvert}$	$g_T = \frac{P_2}{P_1}$
Decibelios (G=−L)	$G_T = 20 \log \frac{\lvert I_2 \rvert}{\lvert I_1 \rvert} [\text{dB}]$	$G_T = 20 \log \frac{\lvert V_2 \rvert}{\lvert V_1 \rvert} [\text{dB}]$	$G_T = 10 \log \frac{P_2}{P_1} [\text{dB}]$
Neperios (G=−L)	$G_T = \ln \frac{\lvert I_2 \rvert}{\lvert I_1 \rvert} [\text{Np}]$	$G_T = \ln \frac{\lvert V_2 \rvert}{\lvert V_1 \rvert} [\text{Np}]$	$G_T = \frac{1}{2} \ln \frac{P_2}{P_1} [\text{Np}]$

- Insertion gain:

	Corriente	Tensión	Potencia
U. naturales (g=1/l)	$g_I = \frac{\lvert I_2 \rvert}{\lvert I_{20} \rvert}$	$g_I = \frac{\lvert V_2 \rvert}{\lvert V_{20} \rvert}$	$g_I = \frac{P_2}{P_{20}}$
Decibelios (G=−L)	$G_I = 20 \log \frac{\lvert I_2 \rvert}{\lvert I_{20} \rvert} [\text{dB}]$	$G_I = 20 \log \frac{\lvert V_2 \rvert}{\lvert V_{20} \rvert} [\text{dB}]$	$G_I = 10 \log \frac{P_2}{P_{20}} [\text{dB}]$
Neperios (G=−L)	$G_I = \ln \frac{\lvert I_2 \rvert}{\lvert I_{20} \rvert} [\text{Np}]$	$G_I = \ln \frac{\lvert V_2 \rvert}{\lvert V_{20} \rvert} [\text{Np}]$	$G_I = \frac{1}{2} \ln \frac{P_2}{P_{20}} [\text{Np}]$

Image parameters of a bilateral Q

Image impedances Z_{01} y Z_{02}



Propagation constant γ

$$\exp(2\gamma) = \left.\frac{V_1 I_1}{V_2 I_2}\right|_{Z_L = Z_{02}}$$

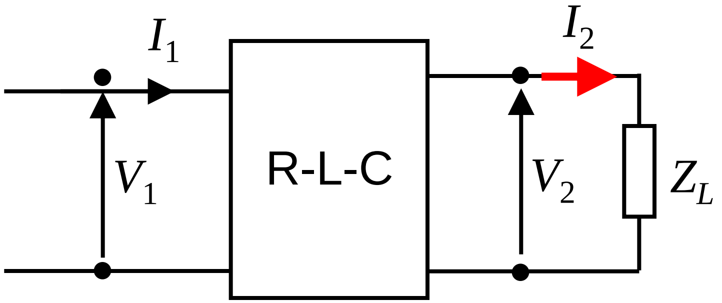


Image parameters for a bilateral and symmetrical Q

Characteristic impedance (Z_0 = Z_{01}=Z_{02}) and **propagation constant (γ)** can be obtained as follows:

$$Z_0 = \sqrt{\frac{B}{C}} = \sqrt{Z_{O.C.} Z_{S.C.}}, \quad \tanh\gamma = \frac{\sqrt{BC}}{A} = \sqrt{\frac{Z_{S.C.}}{Z_{O.C.}}}, \gamma = \frac{1}{2}\ln\frac{1+\tanh\gamma}{1-\tanh\gamma}$$

being $Z_{O.C.}$ y $Z_{S.C.}$ impedances seen trough Q when the ports of the opposite side are respectively open- and short-circuit:

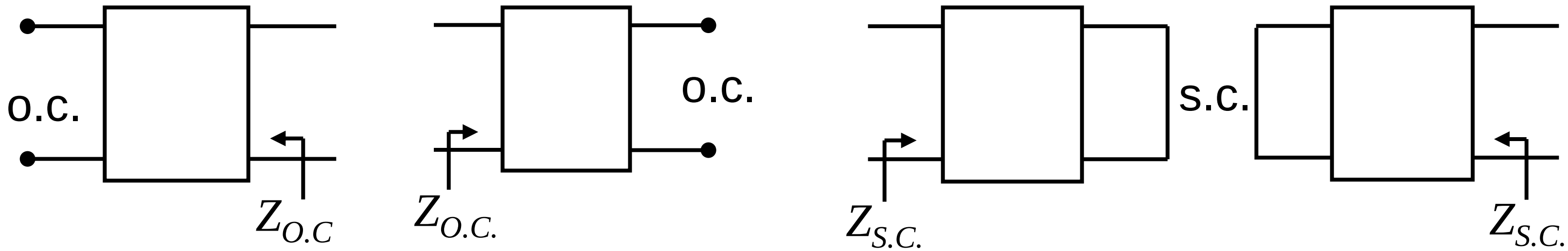
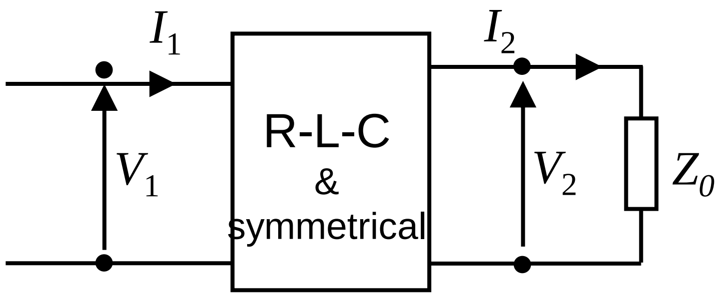


Image parameters for a bilateral and symmetrical Q

Propagation constant γ



$$\exp(\gamma) = \left.\frac{V_1}{V_2}\right|_{Z_L=Z_0} = \left.\frac{I_1}{I_2}\right|_{Z_L=Z_0},$$

$$\gamma = \alpha + j\beta, \quad \beta : \text{phase difference}, \ \alpha : \textbf{transmission loss} (= \text{insertion loss})$$

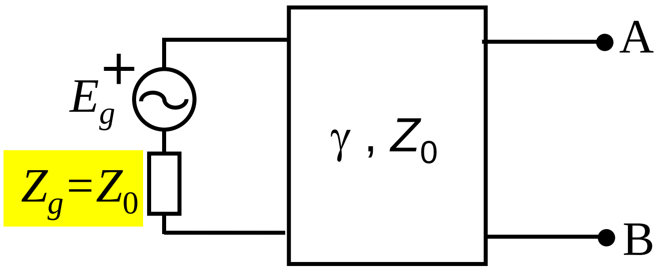
$$\alpha[\text{Np}] = \ln\left|\frac{V_1}{V_2}\right|_{Z_L=Z_0} = \ln\left|\frac{I_1}{I_2}\right|_{Z_L=Z_0} = \frac{1}{2}\ln\left.\frac{P_1}{P_2}\right|_{Z_L=Z_0}$$

$$\alpha[\text{dB}] = 20\log_{10}\left|\frac{V_1}{V_2}\right| = 20\log_{10}\left|\frac{I_1}{I_2}\right| = 10\log_{10}\left|\frac{P_1}{P_2}\right|$$

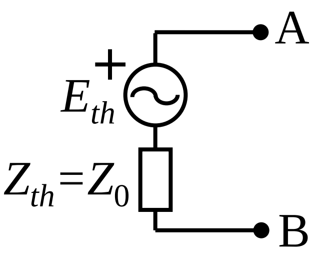
"T" as function of Z_0 and γ

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \cosh\gamma & Z_0 \sinh\gamma \\ \dfrac{1}{Z_0}\sinh\gamma & \cosh\gamma \end{pmatrix}$$

Equivalent Thevenin for a bilateral and symmetrical Q when $Z_g=Z_0$

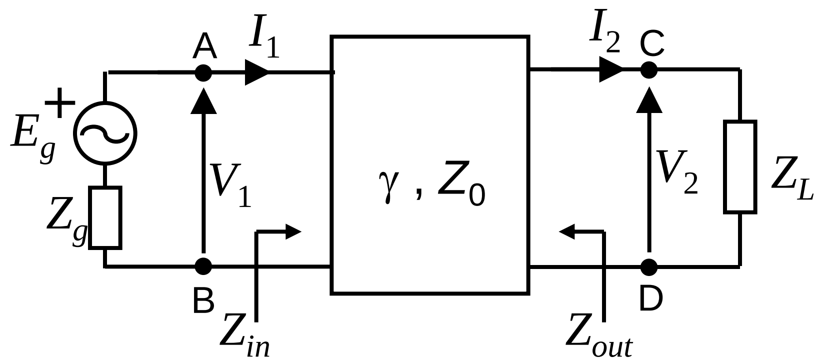


Equivalent Thevenin:

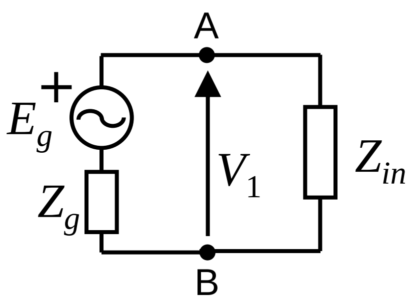


$$E_{th}=\frac{E_g}{CZ_g+A}=\frac{E_g}{\frac{1}{Z_0}\sinh\gamma\cdot Z_0+\cosh\gamma}=E_g\exp(-\gamma)$$

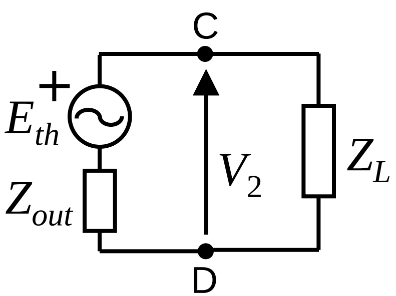
Summarizing, for a bilateral and symmetric quadripole



Equivalent circuits:



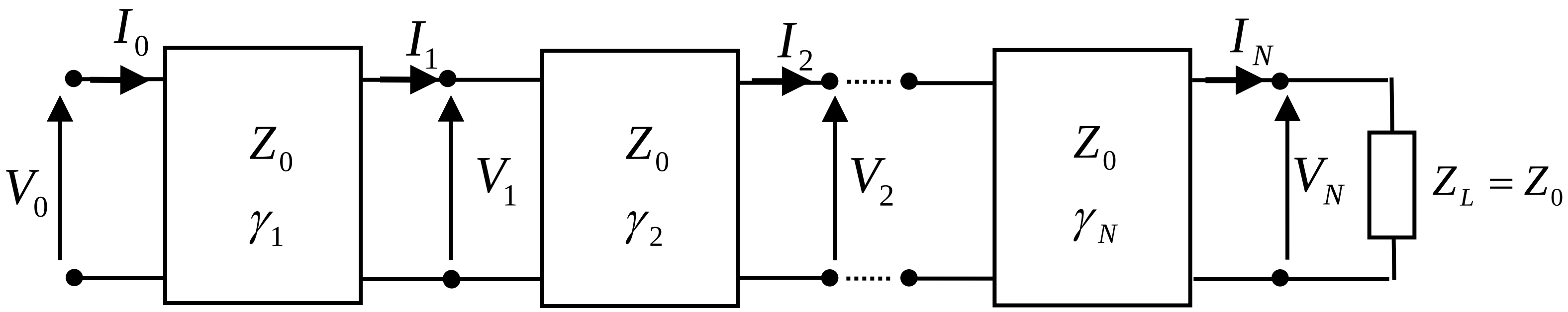
$$\text{if } Z_L = Z_0 \text{ then } \begin{cases} Z_{\text{in}} = Z_0 \\ \exp(\gamma) = \dfrac{V_1}{V_2} = \dfrac{I_1}{I_2} \end{cases}$$



$$\text{if } Z_g = Z_0 \text{ then } \begin{cases} E_{th} = E_g \, e^{-\gamma} \\ Z_{out} = Z_0 \end{cases}$$

Cascaded association using image parameters

***N* bilateral and symmetric quadripoles** (all with the same Z_0) **connected in cascade:**



$$V_N = V_{N-1} e^{-\gamma_N} = V_{N-2} e^{-\gamma_{N-1}} e^{-\gamma_N} = \ldots = V_0 e^{-(\gamma_1+\gamma_2+\ldots+\gamma_N)}$$

$$I_N = I_0 e^{-(\gamma_1+\gamma_2+\ldots+\gamma_N)}$$