U5. Resonant Circuits

Circuit Analysis, GITT
Curso 2016-2017

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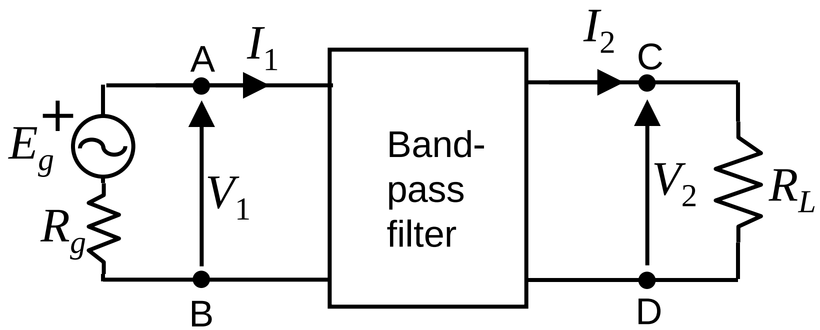
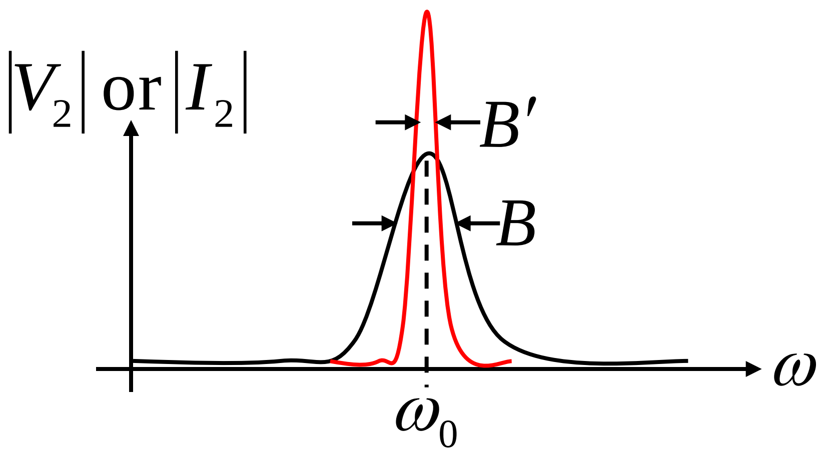
Introduction

- Resonant circuits are band-pass filters



- ω_0: Resonance frequency
- B: Bandwidth
- Q: Quality factor, $Q = \frac{\omega_0}{B_{3\mathrm{dB}}}$

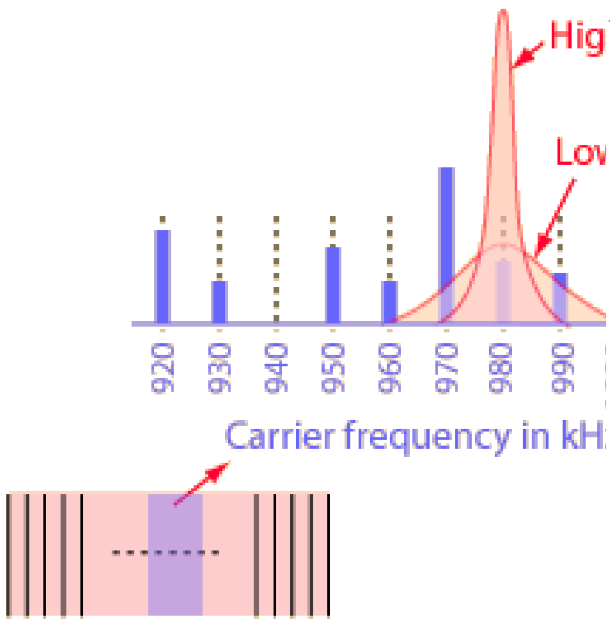
$$Q' = \frac{\omega_0}{B'_{3\mathrm{dB}}} > Q = \frac{\omega_0}{B_{3\mathrm{dB}}}$$



The higher Q the more selective is the circuit

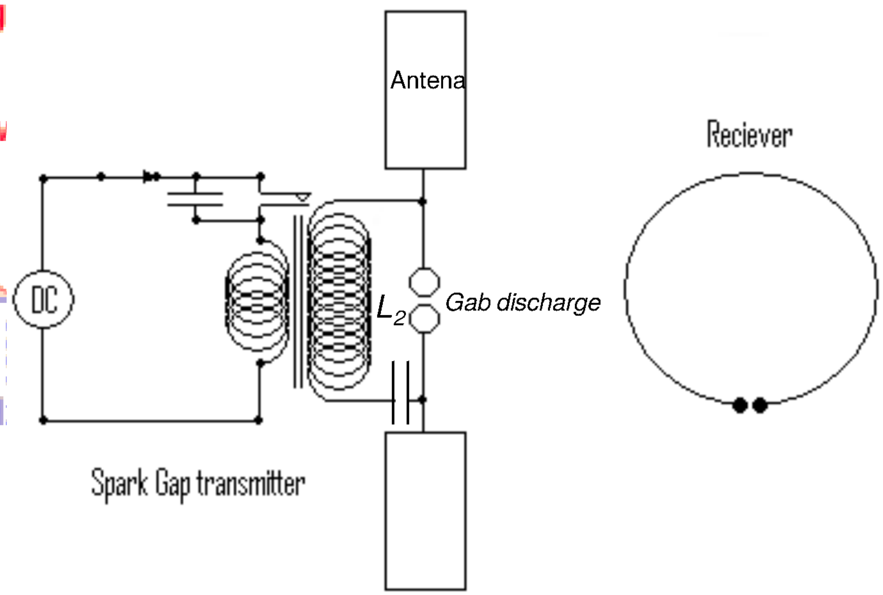
Introduction

- Resonant circuits select signals of different frequencies (http://hyperphysics.phy-astr.gsu.edu/hbase/electric/serres.html)



10 kHz bandwidth from 540-1600 kHz for 106 possible bands

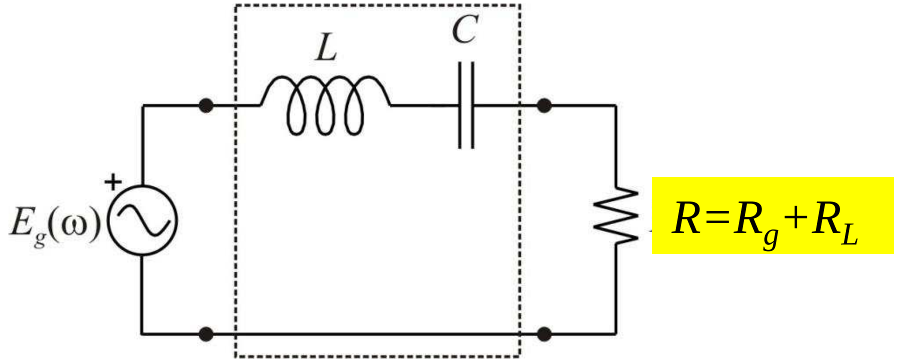
AM Radio



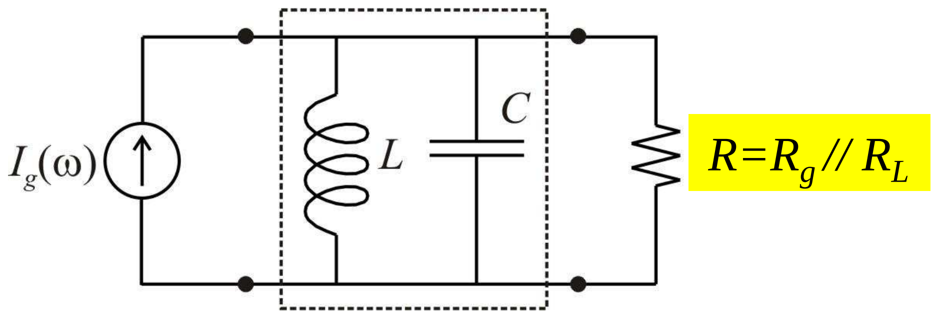
1887 experimental setup of Hertz's apparatus, **spark-gap transmitter**: *C* is charged and then "abruptly" discharged with a under-damped oscillation with frequency ω_0.
(http://en.wikipedia.org/wiki/Spark-gap_transmitter)

Examples of resonant circuits

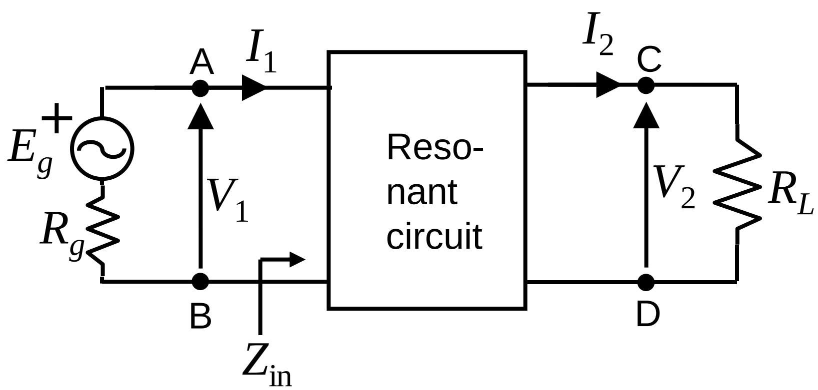
- RLC-Serial



- RLC-Parallel



Resonance frequency (ω_0)



- It is the frequency for which:
  - $|I_2|$ and $|V_2|$ **becomes maximum**
  - The input impedance, Z_{in}, **becomes real** since the rective inductance and capacitance cancels each other:

$$\omega L\Big|_{\omega=\omega_0} = \frac{1}{\omega C}\Big|_{\omega=\omega_0} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

  - The stored energy in both elements *C* and *L* **is the same and maximum** (the stored energy oscillates between *C* and *L*).

Quality factor (*Q*)

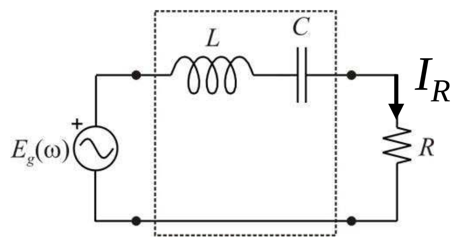
- It is the ratio of two energies:

$$Q = 2\pi \frac{\text{Maximum amount of stored energy (in } C, L)}{\text{Energy lost in a period (in } R)} = \omega_0 \frac{W_{\max}}{P_R}$$

(The maximum stored energy produces at $\omega = \omega_0$)

- Q of a RLC-series circuit:

$$Q_S = \frac{\omega_0 L}{R} = \frac{1}{R\omega_0 C}$$

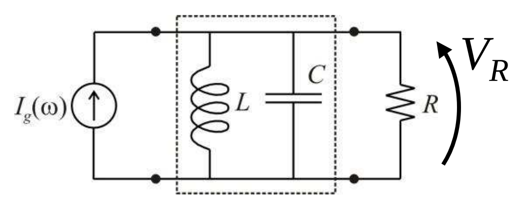


$$W_{\max} = \frac{1}{2} L |I_R|^2$$

$$P_R = \frac{1}{2} |I_R|^2 R$$

- Q of a RLC-parallel circuit:

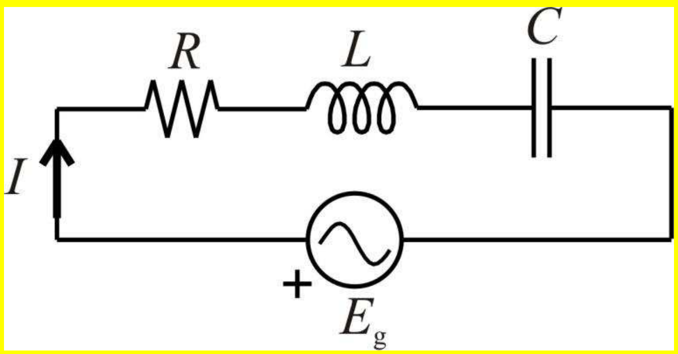
$$Q_P = R\omega_0 C = \frac{R}{\omega_0 L}$$



$$W_{\max} = \frac{1}{2} C |V_R|^2$$

$$P_R = \frac{1}{2} \frac{|V_R|^2}{R}$$

RLC-Series

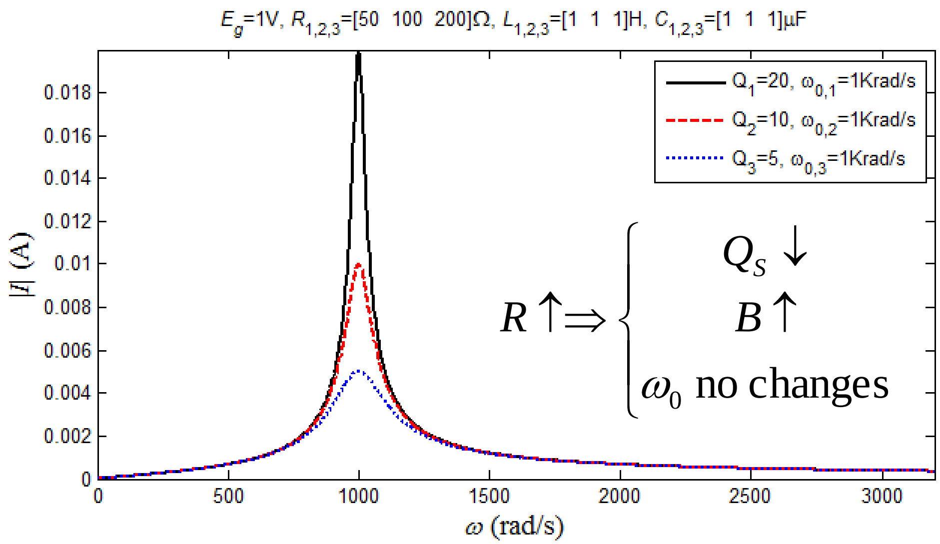


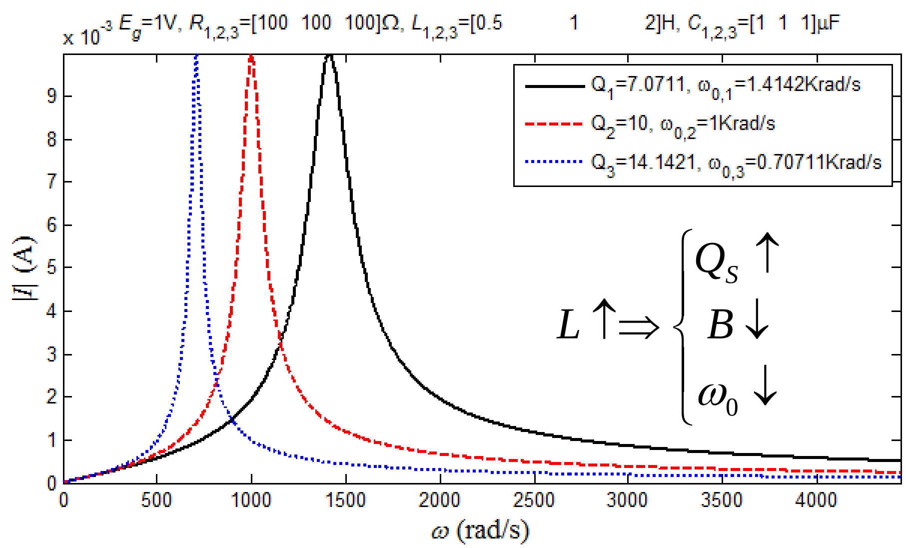
$$RLC-\text{series}: Q_S = \frac{\omega_0 L}{R} = \frac{1}{R\omega_0 C}$$

$$I = \frac{E_g}{R + j\left(\omega L - \frac{1}{\omega C}\right)} = \frac{I_{\max}}{1 + jQ_S\delta\left(\frac{\delta+2}{\delta+1}\right)}$$

$$I \cong \frac{I_{\max}}{1 + j2Q_S\delta} \text{ for } \delta << 1,$$

$$\delta = \frac{\omega - \omega_0}{\omega_0}, \quad I_{\max} = \frac{E_g}{R}, \quad \omega_0 = \frac{1}{\sqrt{LC}}.$$





RLC-Series

$$Q_S = \frac{\omega_0 L}{R} = \frac{1}{R\omega_0 C},$$

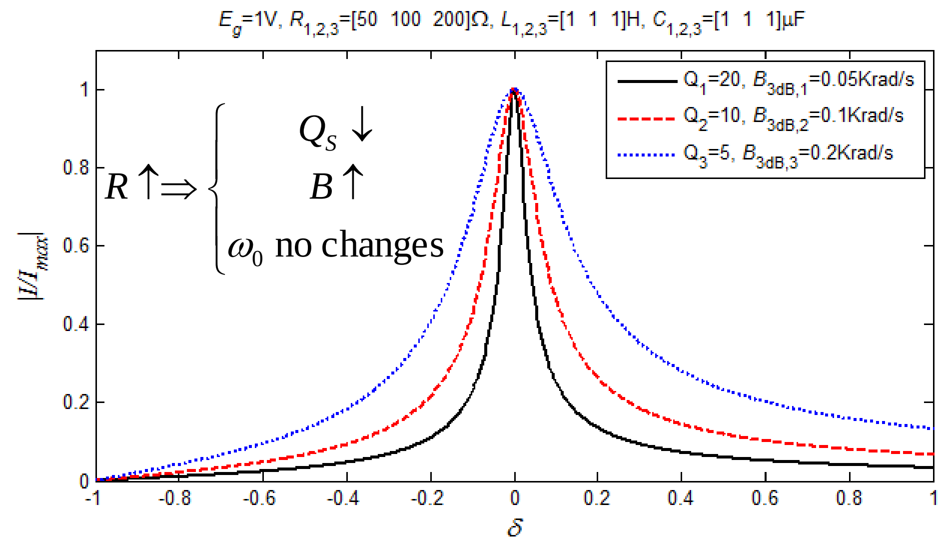
$$\left|\frac{I}{I_{\max}}\right| = \frac{1}{\sqrt{1+\left(Q_S\delta\frac{\delta+2}{\delta+1}\right)^2}},$$

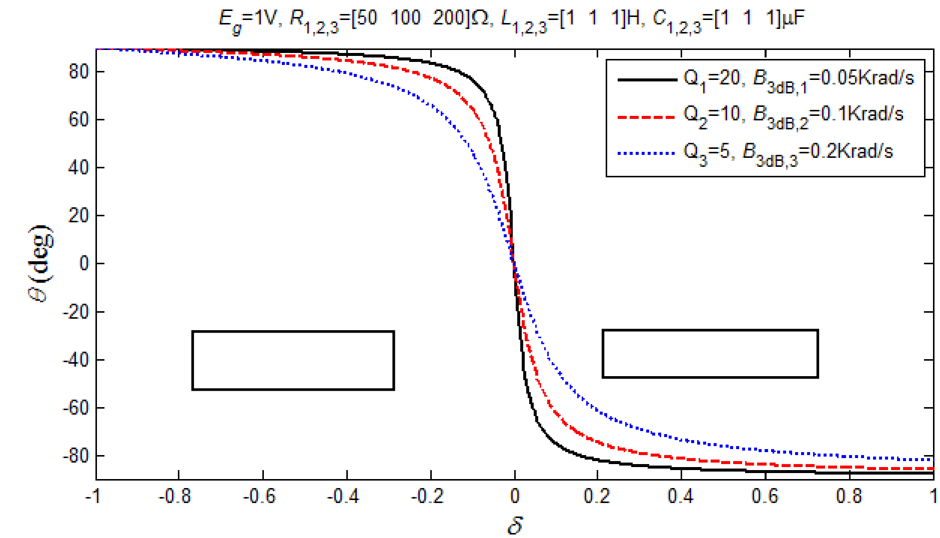
$$\theta = -\arctan Q_S\delta\frac{\delta+2}{\delta+1},$$

for $\delta << 1$ and $Q_S \geq 10$:

$$\left|\frac{I}{I_{\max}}\right| \cong \frac{1}{\sqrt{1+(2Q_S\delta)^2}},$$

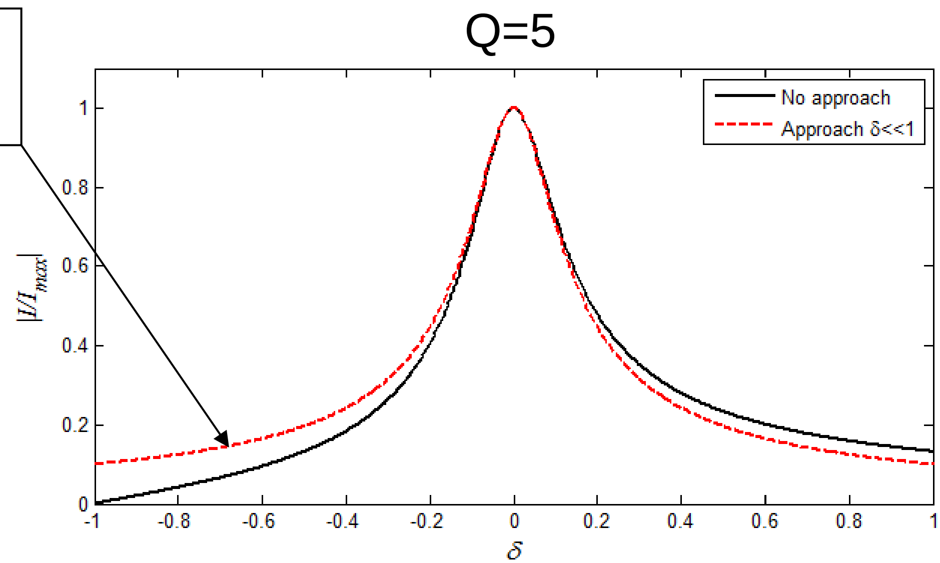
$$\theta \cong -\arctan 2Q_S\delta.$$

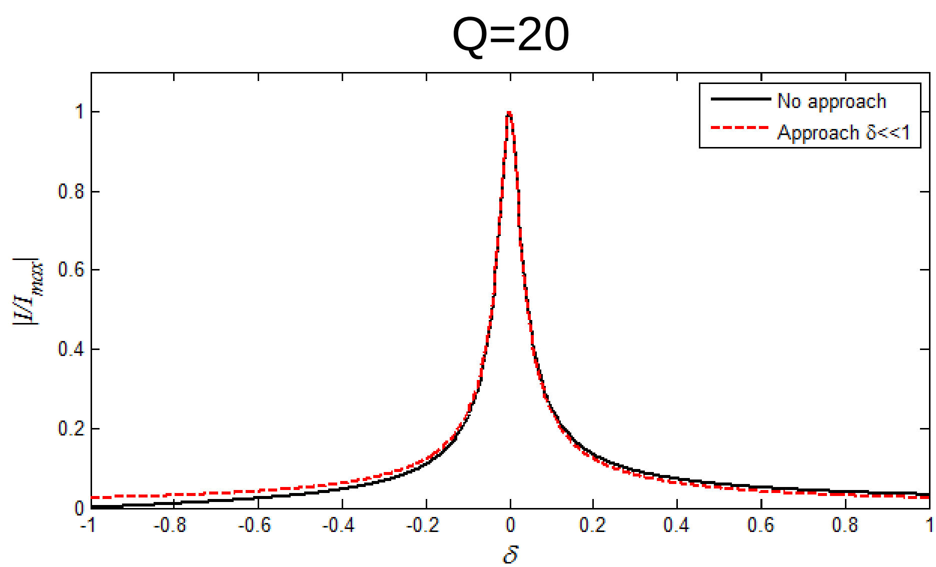




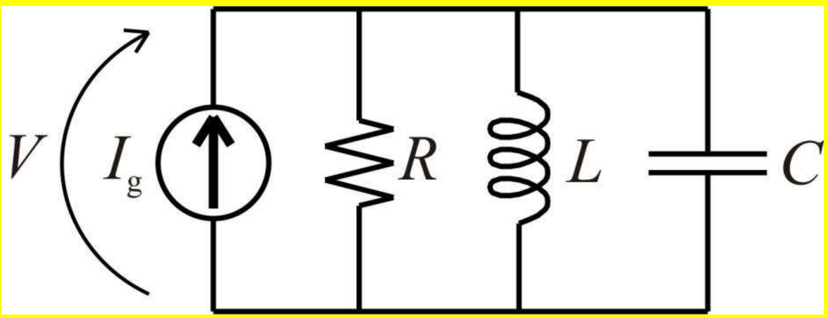
RLC-Series

Symmetric around δ=0





RLC-Parallel

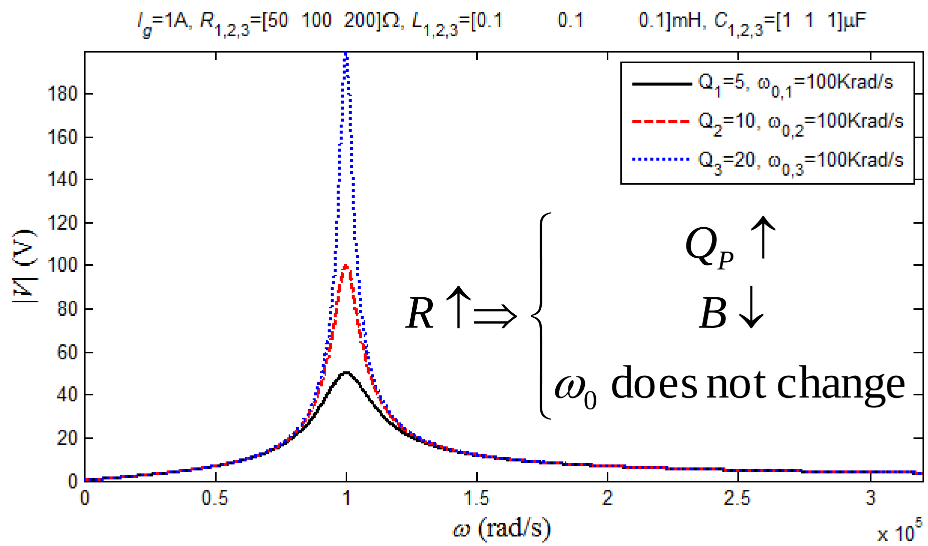


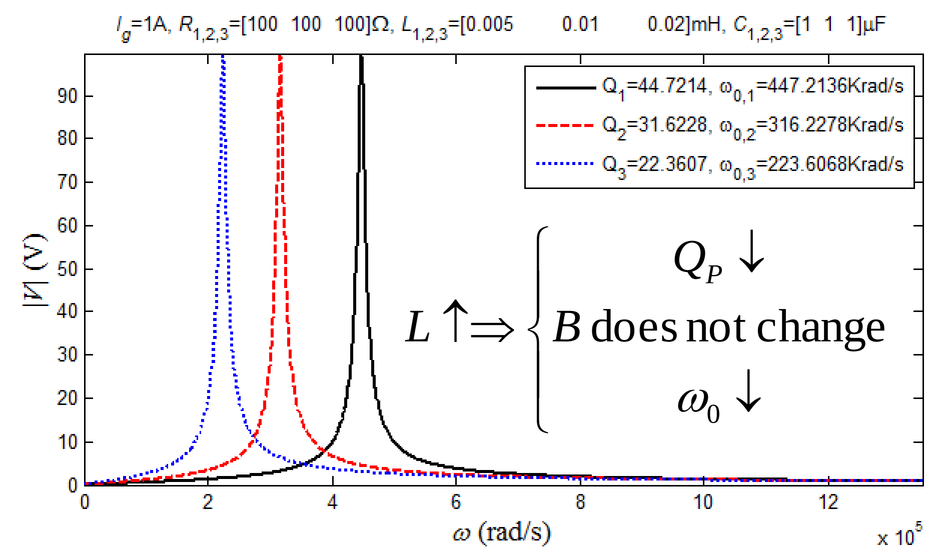
$$RLC-\text{parallel}: Q_P = \frac{R}{\omega_0 L} = R\omega_0 C$$

$$V = \frac{I_g}{\frac{1}{R} + \mathrm{j}\left(\omega C - \frac{1}{\omega L}\right)} = \frac{V_{\max}}{1 + \mathrm{j}Q_P\delta\left(\frac{\delta+2}{\delta+1}\right)},$$

$$\delta = \frac{\omega - \omega_0}{\omega_0}, \quad V_{\max} = I_g R, \quad \omega_0 = \frac{1}{\sqrt{LC}}.$$

$$V \cong \frac{V_{\max}}{1 + \mathrm{j}2Q_P\delta} \text{ for } \delta << 1.$$





Bandwidth ($\delta \ll 1$, $Q \geq 10$)

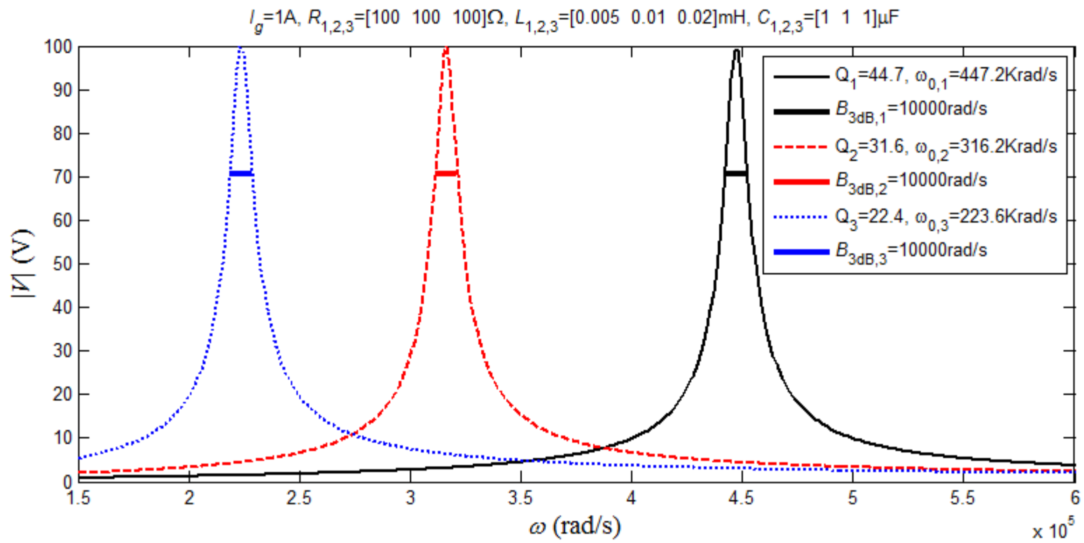
$$B_{XdB} = \omega_2 - \omega_1,$$
$$\omega_2 = \omega_0 \left(1 + \delta_C\right),$$
$$\omega_1 = \omega_0 \left(1 - \delta_C\right),$$
$$\delta_C = \frac{1}{2Q}\sqrt{10^{\frac{XdB}{10}} - 1}.$$

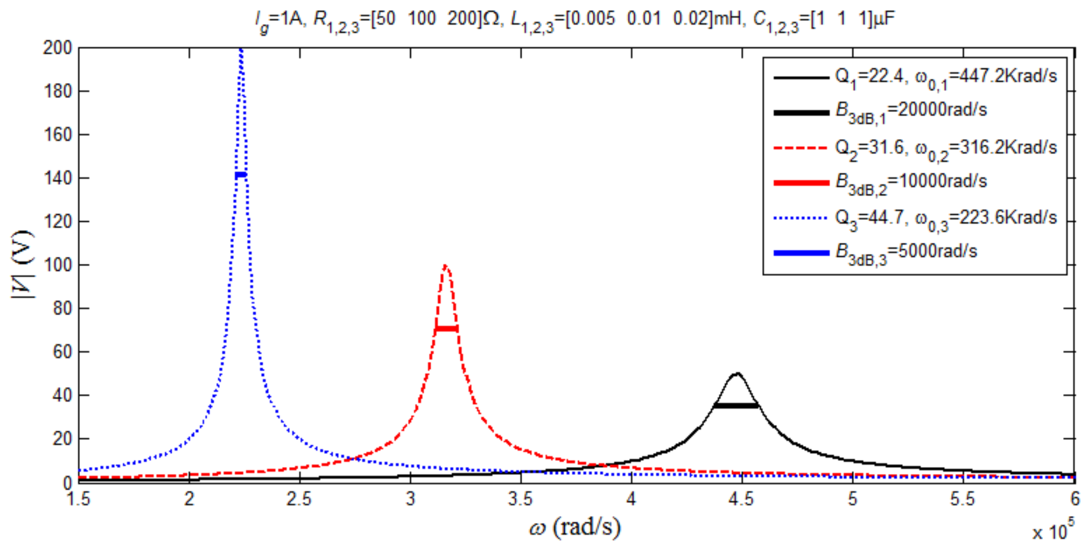
for $X = 3dB$:

$$B_{3dB} = \frac{\omega_0}{Q}.$$

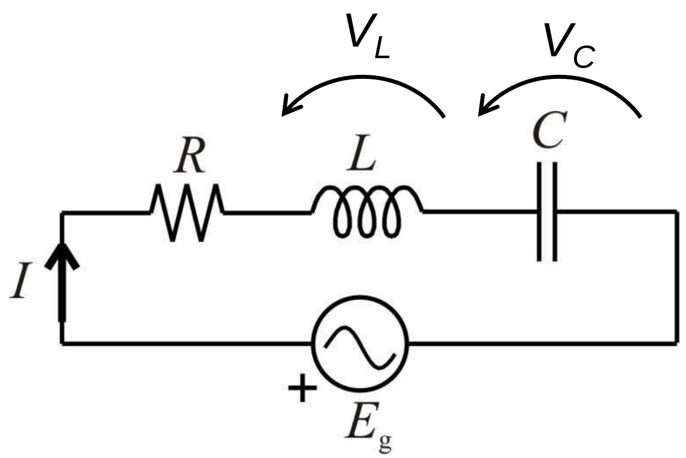
$$B_{3dB}\big|_P = \frac{1}{RC}, \quad B_{3dB}\big|_S = \frac{R}{L}$$

For example RLC-parallel:





Over-voltage factor (Q_S)



$$V_L = \mathrm{j}\omega L I$$

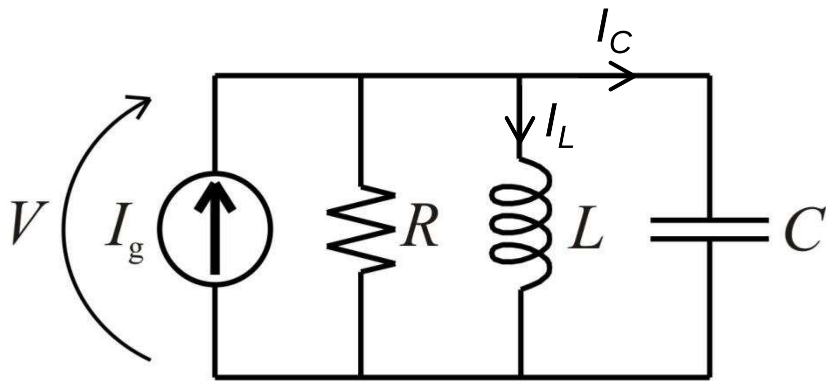
$$V_C = -\mathrm{j}\frac{1}{\omega C} I$$

$$\text{for } \omega = \omega_0 : \quad I = I_{\max} = \frac{E_g}{R}, \; V_L = \mathrm{j}\frac{\omega_0 L}{R} E_g$$

$$V_C = -\mathrm{j}\frac{1}{\omega_0 C R} E_g$$

$$V_L = -V_C, \; |V_L| = |V_C| = Q_S E_g$$

Over-current factor (Q_P)



$$I_L = -\mathrm{j}\frac{1}{\omega L}V$$

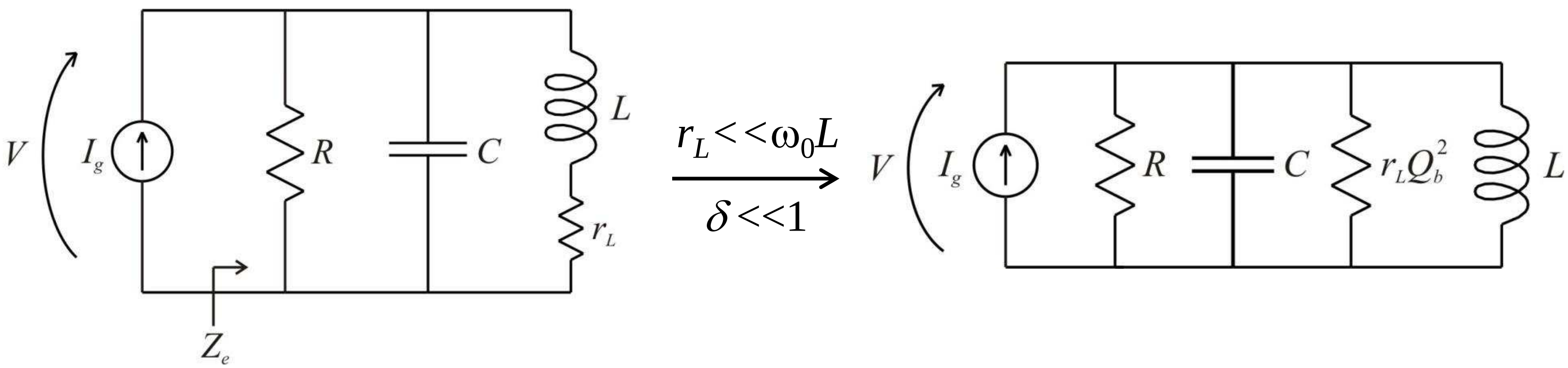
$$I_C = \mathrm{j}\omega C V$$

$$\text{for } \omega = \omega_0 : \quad V = V_{\max} = R I_g , \; I_L = -\mathrm{j}\frac{R}{\omega_0 L} I_g$$

$$I_C = \mathrm{j}\omega_0 C R I_g$$

$$I_L = -I_C , \; |I_L| = |I_C| = Q_P I_g$$

Real RLC-Parallel



$$\frac{1}{Z_e} = \frac{1}{R} + \mathrm{j}\omega C + \frac{1}{r_L + \mathrm{j}\omega L},$$

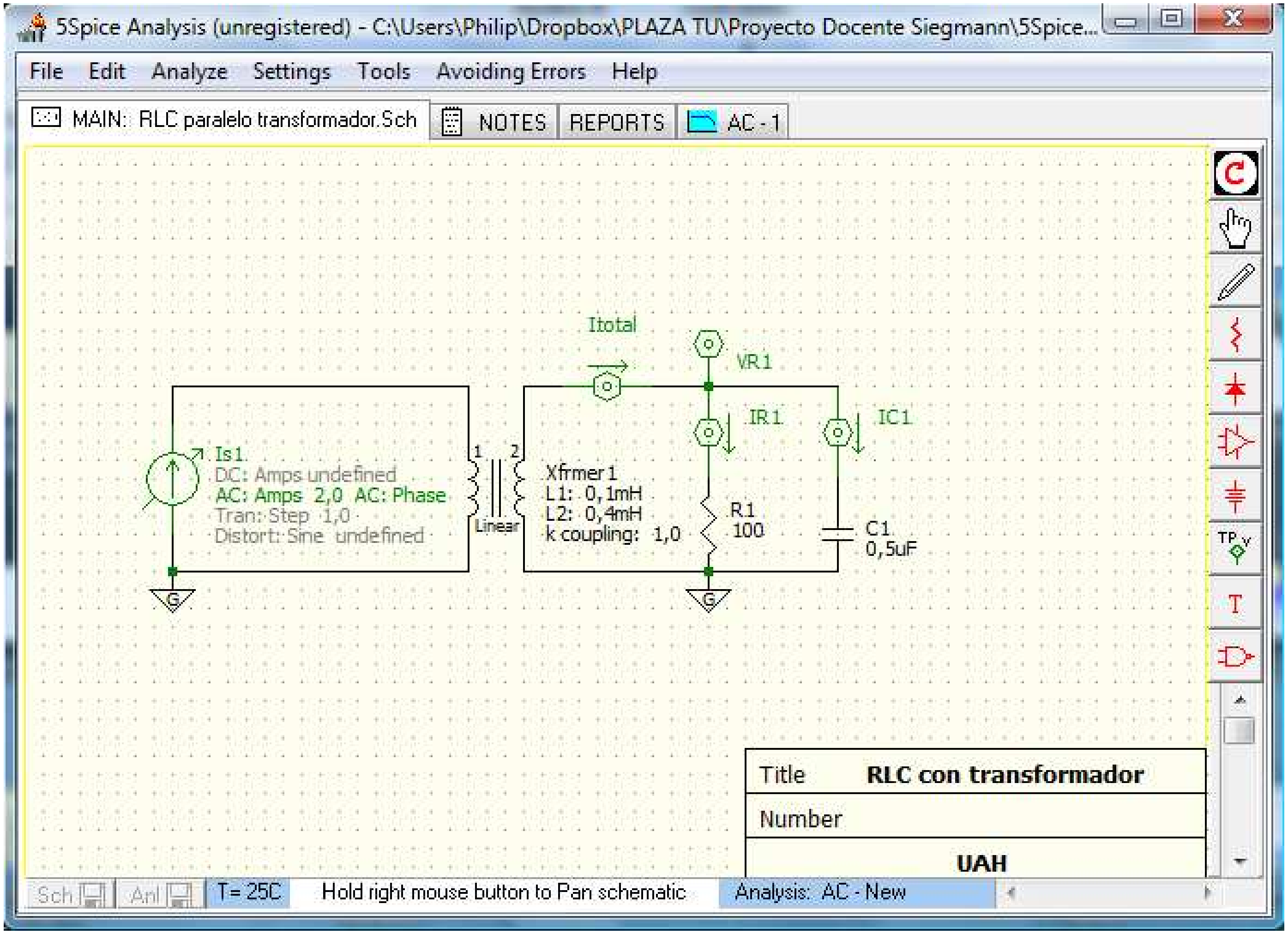
$$\frac{1}{r_L + \mathrm{j}\omega L} \approx \frac{1}{r_L Q_b^2} - \mathrm{j}\frac{1}{\omega L} \quad \text{si} \quad Q_b = \frac{\omega_0 L}{r_L} >> 1 \,\&\, \delta << 1$$

$$\frac{1}{Z_e} \approx \frac{1}{R} + \mathrm{j}\omega C + \frac{1}{r_L Q_b^2} - \mathrm{j}\frac{1}{\omega L} \quad \text{parallel association } (R, C, r_L Q_b^2, L)$$

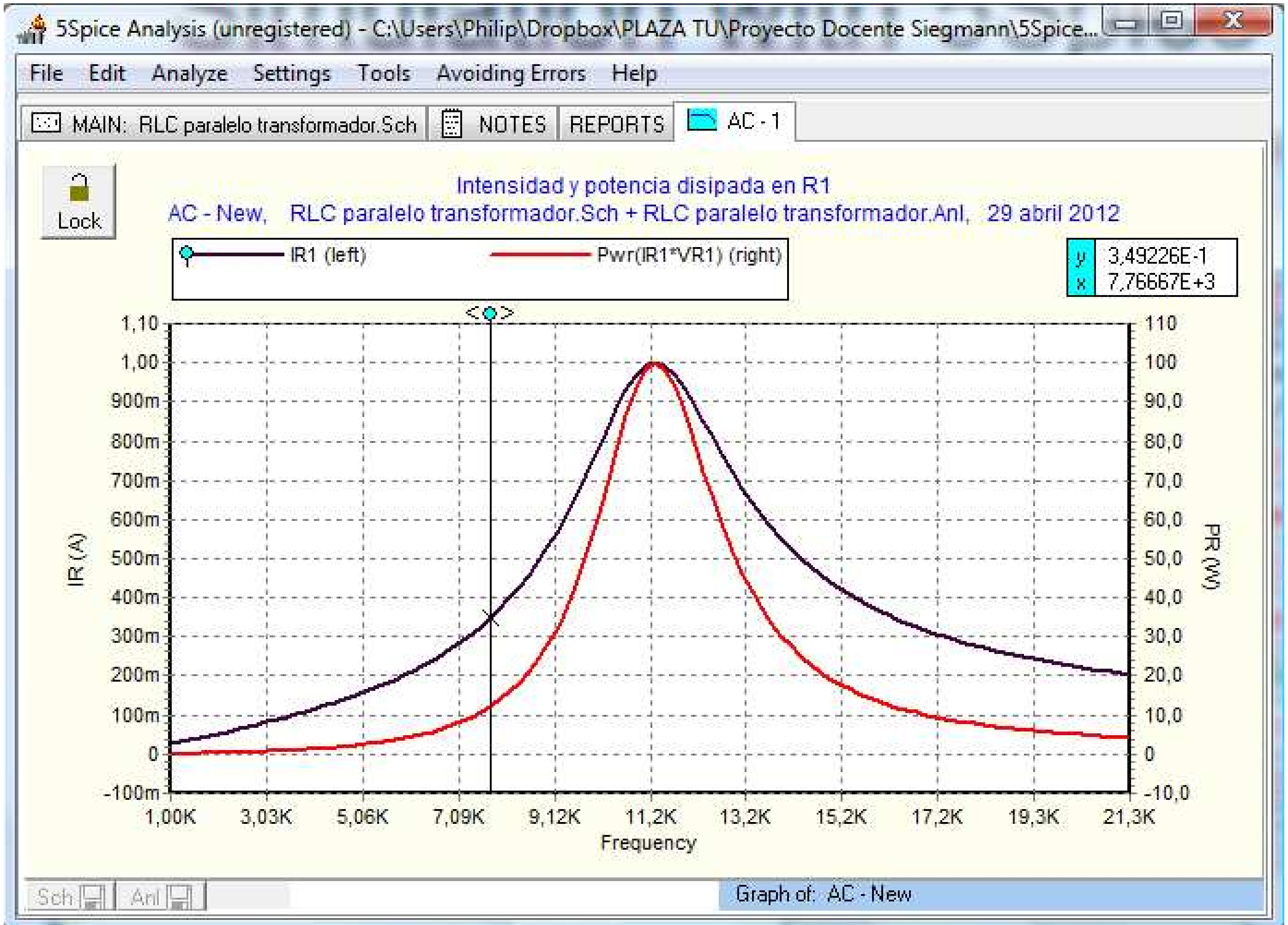
$$R_T = \frac{R r_L Q_b^2}{R + r_L Q_b^2}$$

Quality factor: $Q = R_T \omega_0 C$

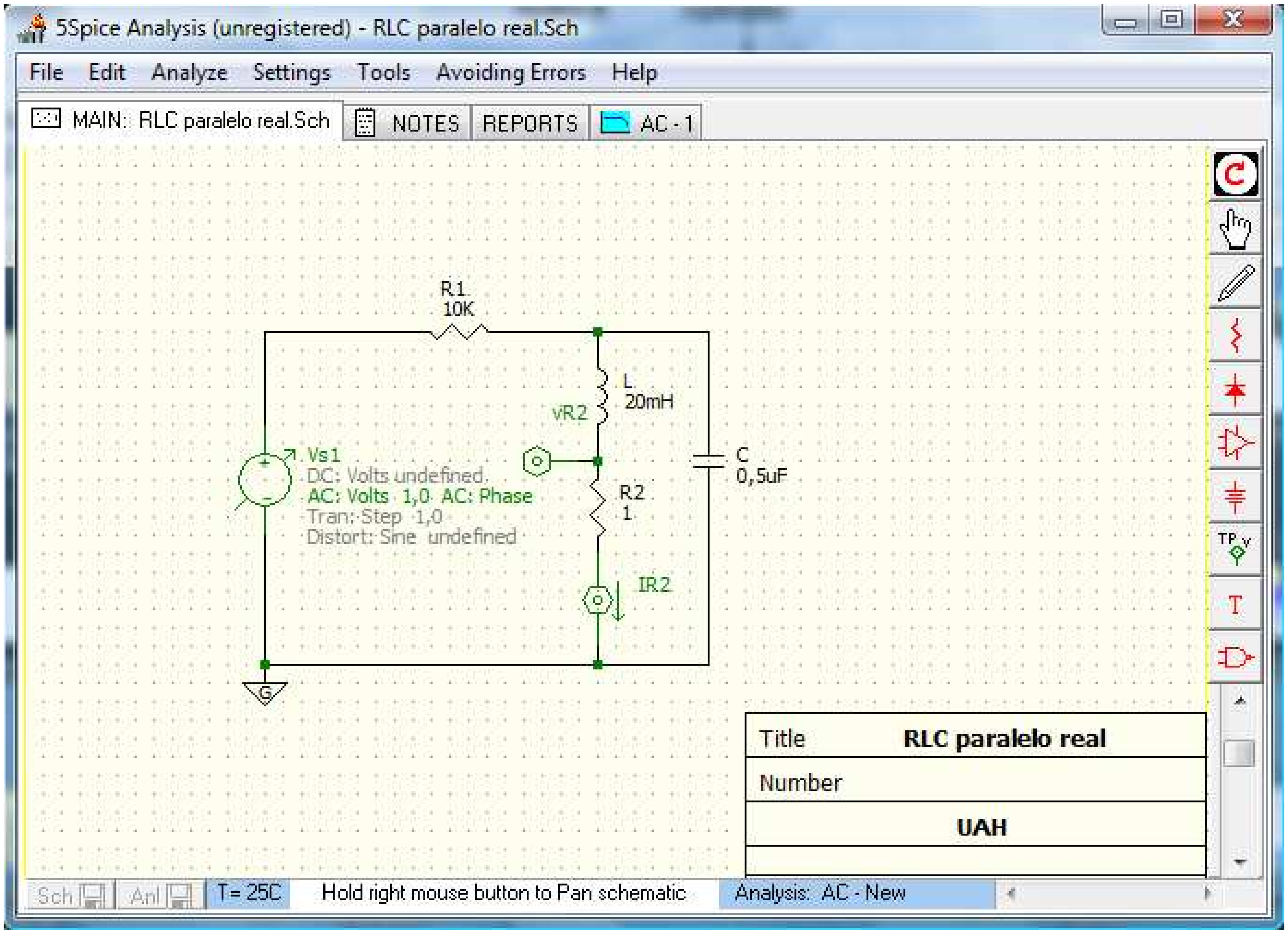
Simulation with 5Spice



Simulatio with 5Spice



Simulation with 5Spice



Simulation with 5Spice