Module 1. Basic concepts and fundamental laws of electric circuits

Circuit Theory, GIT, 2018-19

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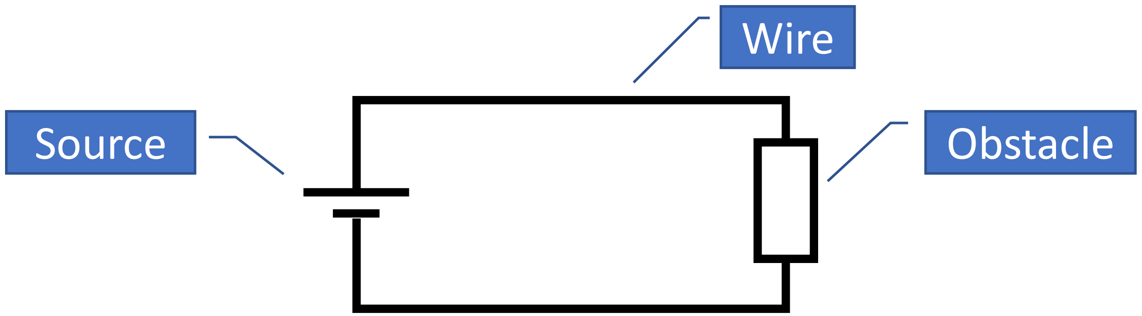
Outline

- **The electrical circuit**
- **The movement of the charges**
- **The electrical current**
- **The voltage**
- **Ground connection**
- **Ohm's law**
- **Energy sources**
- **Voltage difference determination**
- **Electrical Power**
- **Structural parts of a circuit**
- **Kirchhoff's laws**
- **Passive elements: Resistor, Capacitor and Inductor**
- **Equivalent resistance**

The electrical circuit

- Basically, by studying electrical circuits we are studying a way of transmitting energy (e.g. information) from one place to another.
- The energy is transmitted by the movement of electrical charges.
  - The charge movement is produced by energy sources
  - The charges move along a path which will be the perfectly conducting wires.
  - They move trough obstacles, like **resistances** (who dissipate the energy) and other elements as **capacitors** and **inductors** who are able to store energy and deliver it.
- Representation:



Model used for the movement of the charges

- The movement of electrical charges will be along wires and through the different elements.
- The electrical charges are already present in the wires, **no new charges are supplied!!**
- Their movement follow the next three simple rules:
  - **Neutrality condition:** as many positive as negative charges at any time and place.
  - Unison movement: as a consequence of the neutrality al charges will move at once.
  - It is supposed that the **positive charges, *q+*,** are moving (instead of the negative electrons).

The electrical current

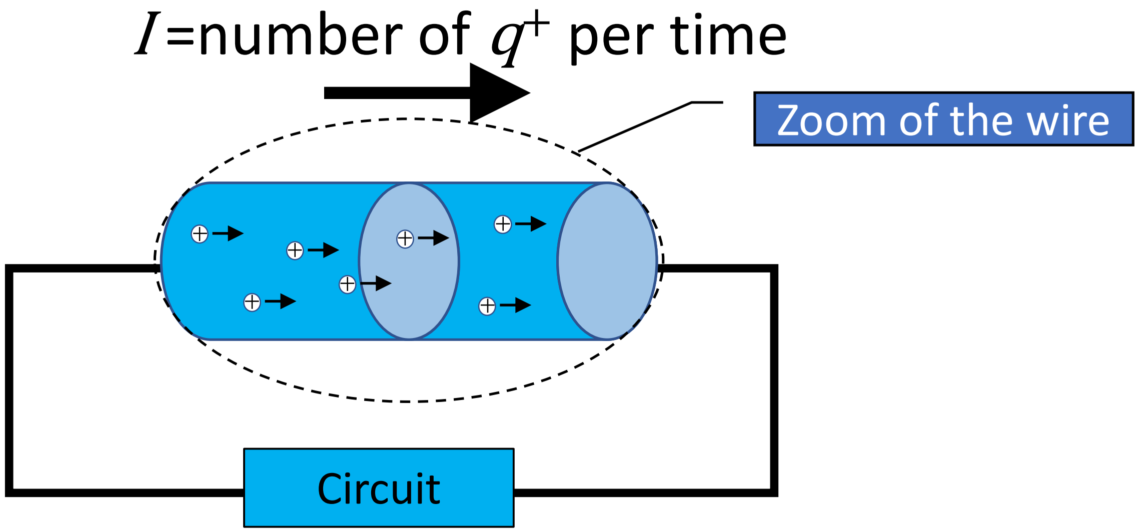
- The magnitude used to measure the movement of the positive charges is the electrical **current** or **intensity**

$$i(t) = \frac{dq}{dt}$$

- with unit in **Amperes** (A), A=C/s. It is the variation of the charges per unit of time. The unit of the charges is the Coulomb (C), one Coulomb corresponds to the charge of 1,6·10^19 electrons.
- If there is no change of the charges with the time then $i(t) = 0$.
- If the charge variation is constant, the corresponding constant current will be denoted with capital letter: $i(t) = constant = I$.

The electrical current

- The electrical current in a wire measures the number of *elemental charges* that crosses a normal surface to the wire in a time instant.
- A movement **direction has to be assigned** to the current which is, by convention, the **direction that follows the positive charges**



The voltage

- The voltage is the energy at the instant t, $w(t)$, per unit of positive charge

$$v(t) = \frac{dw(t)}{dq},$$

- with units in **Volts** (V), V=Joule/Coulomb.
- If the energy of the charge does not change with the time or if the voltage value of a charge is constant we will use capital letter: $v(t) = constant = V$.
- The voltage has different interpretations depending on what is measured.

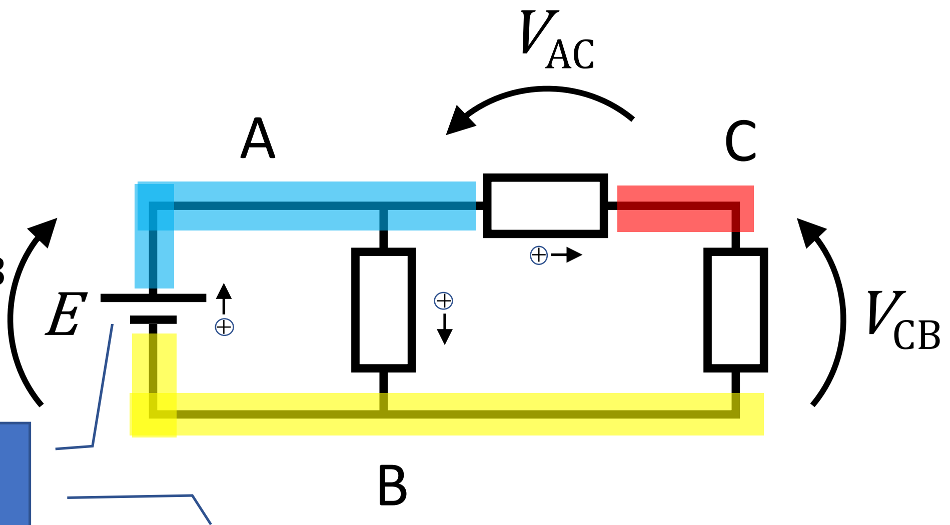
Type of voltages

- Voltage as the energy delivered by a source to the unit of positive charge $q+$: **Electromotive Force,** $e(t)$ or emf.
- Voltage as the potential energy of the $q+$ for being at a specific place A (within the circuit): **Voltage** or **Potential at A**, $v_A(t)$.
- Voltage difference as the energy difference experimented by a $q+$ when moving through the elements of the circuit (i.e from B to A): **Potential** or **Voltage difference**, $v_{AB}(t) = v_A(t)$- $v_B(t)$

Type of voltages

- The voltage difference and the electromotive force (emf) are indicated on the figure of the circuit using an arrow pointing in the direction where it is supposed to have the higher value of potential

$$E = V_{AB} = V_{AC} + V_{CB}$$



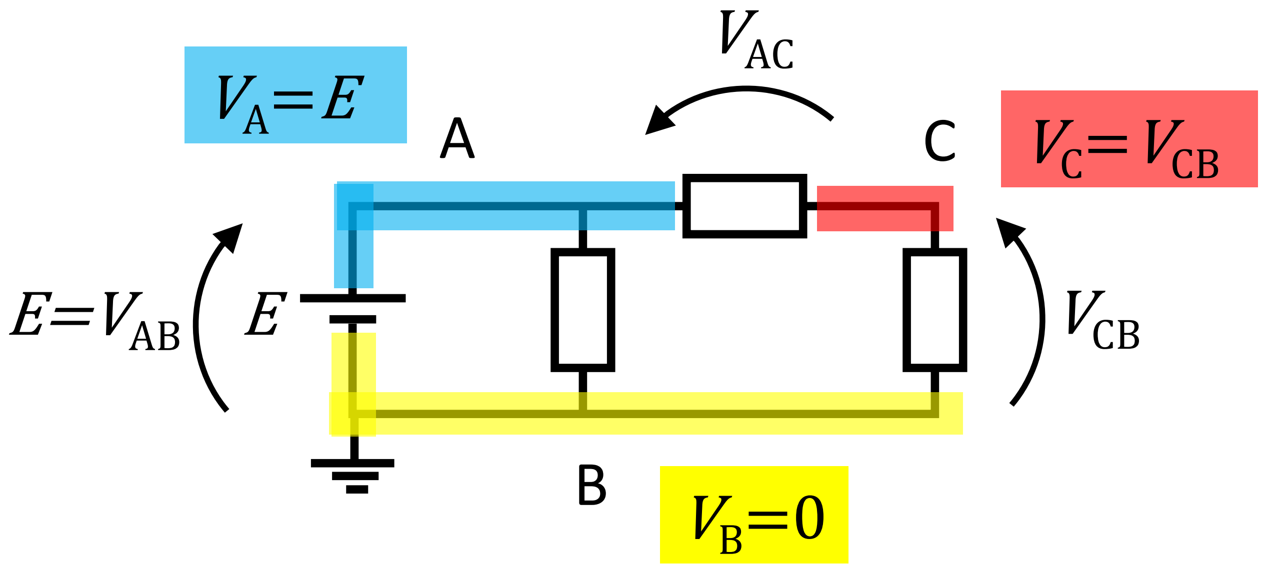
- 3 Potential levels: $\{V_A, V_B, V_C\}$
- 3 Voltage differences: $\{V_{AB}, V_{AC}, V_{CB}\}$
- One emf: $E = V_{AB}$

The source provides $q+$ with energy E when it moves trough it from B to A

The same energy is lost when $q+$ moves back from A to B through any other way, dissipating $V_{AB} = E$ when passing through the different elements

Ground connection

- The ground connection (GND) is a wire connected to the “Earth” that makes the value of the potential to be zero at the place were it is connected.
- The symbol is ⏚ and allows to obtain the potential levels at the different places:



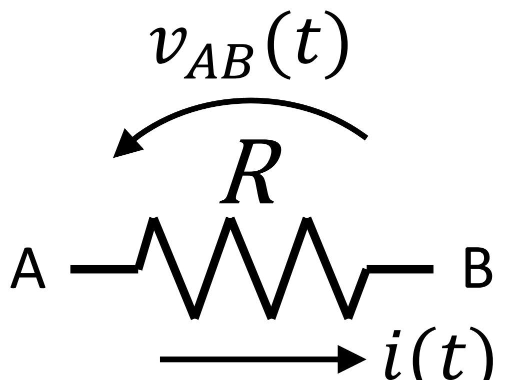
Ohm's law

- Ohm empirically discovered that the relation between the voltage difference applied to an electrical element and the corresponding generated current trough it is a *constant value*, R, and electrically characterizes this element

$$R = \frac{v_{AB}(t)}{i(t)},$$

It is called ***Resistance*** with units in Ohm's, (Ω), Ω =V/A. It measures how bad the charges moves trough the element.

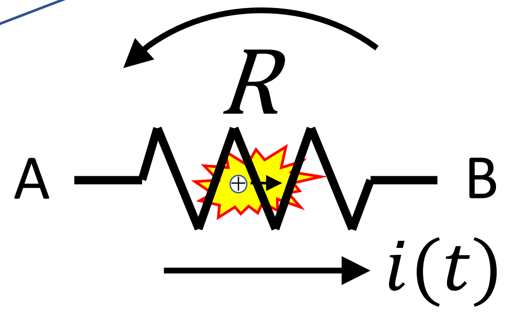
Representation:



Ohm's law

- The voltage difference at an element with resistance, R, can be obtained from its resistance and the amount of charges that move through it. It measures the energy dissipated by the unit of charge when crossing this element. For any instant t :

$$v_{AB}(t) = Ri(t)$$



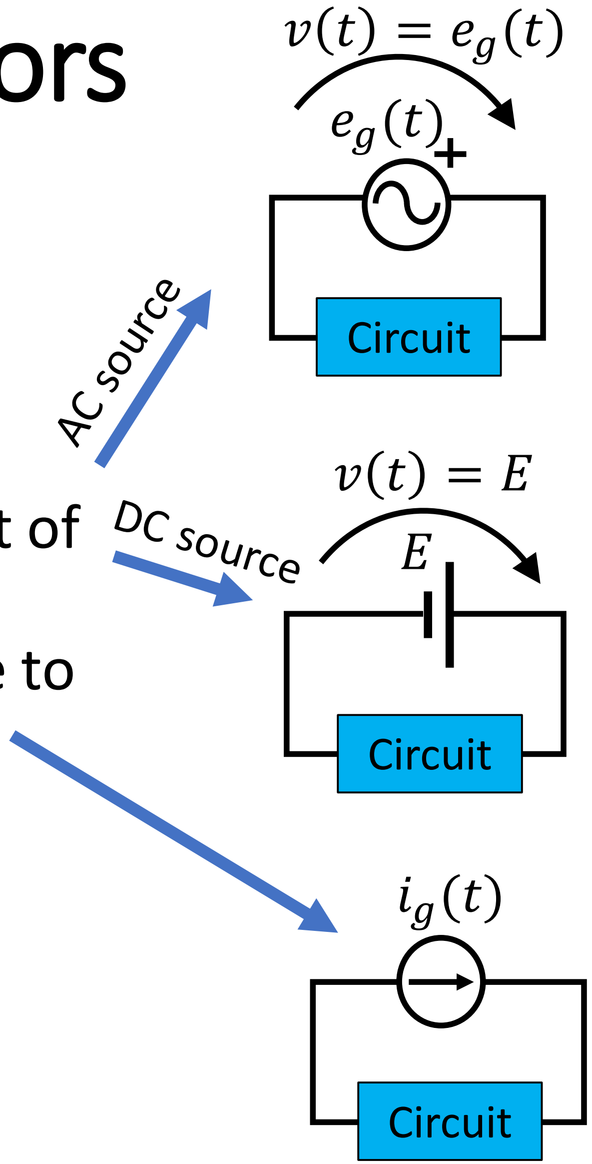
For the indicated voltage difference the current has the corresponding indicated direction:

The q+ is supposed to move from higher to lower potential: from A to B

If the voltage difference arrow and the current arrow are pointing in the same direction then the sign has to be negative in Ohm’s equation

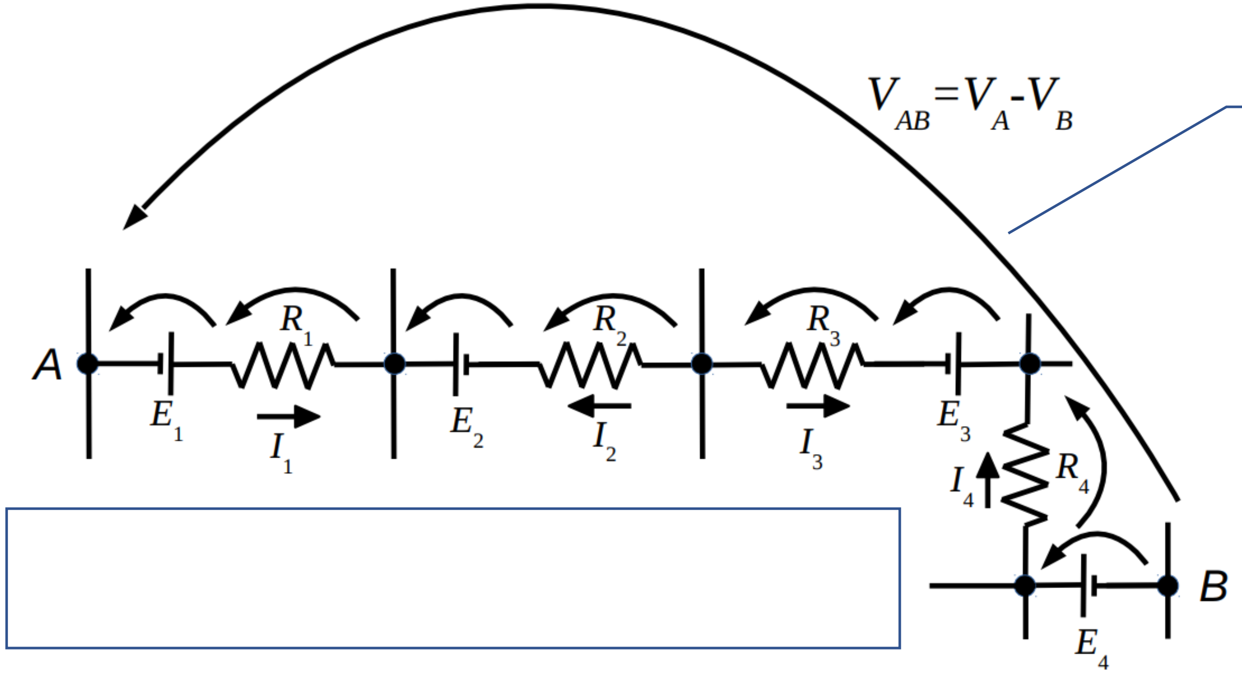
Energy sources or generators

- The resistances dissipates the energy of the charges that passes trough them.
- The energy is provided by the sources
  - **Voltage source**, $e_g(t)$, that gives a certain energy per unit of positive charge to make it move
  - **Current sources**, $i_g(t)$, that applies the necessary voltage to maintain a certain current.
- The sources can also dissipate energy if they act in opposite direction to the actual charge movement.
- If charges do not move then the energy is not transferred/dissipated



Voltage difference determination

- When calculating the voltage difference between two points A and B in a network
  - It is **independent on the path** you choose between A and B
  - Add all the voltage differences at each element along the selected path
  - The voltage difference at a **voltage source is independent of the current**
  - For the voltage difference at a resistance apply **Ohm's law** taking into account the relative directions of the voltage and the current (supposing the currents through all *R* are known)



It is recommended to draw the arrow and all the subarrows of the voltage differences at each element

Electrical Power

- The power at an instant *t*, *p(t)*, is the energy provided/dissipated per time

$$p(t) = \frac{dw}{dt} = \frac{dw}{dq}\frac{dq}{dt} = v(t)i(t).$$

- for a constant energy supply, also known as **Direct Current** (**DC**), the power is a constant (use capital letter)

$$P = VI,$$

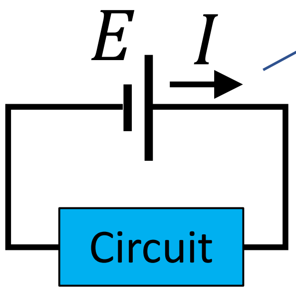
With units Watt (W), W=Jouls/scond=Volt·Ampere

Power sign convention

- The power of a resistance is always dissipated and $\geq$ **0**:

$$P_R = VI = RI^2$$

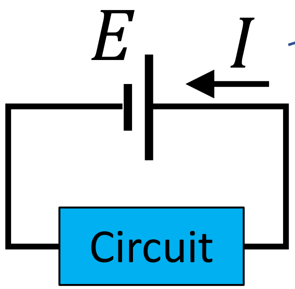
- The power of a DC voltage source:
  - It is **>0** if delivered by the source



$$P_E = EI$$

The source, *E*, pushes the charges in the direction in which they are flowing providing them with energy which is then transferred to the rest of the circuit

  - It is **<0** if dissipated or absorbed by the source



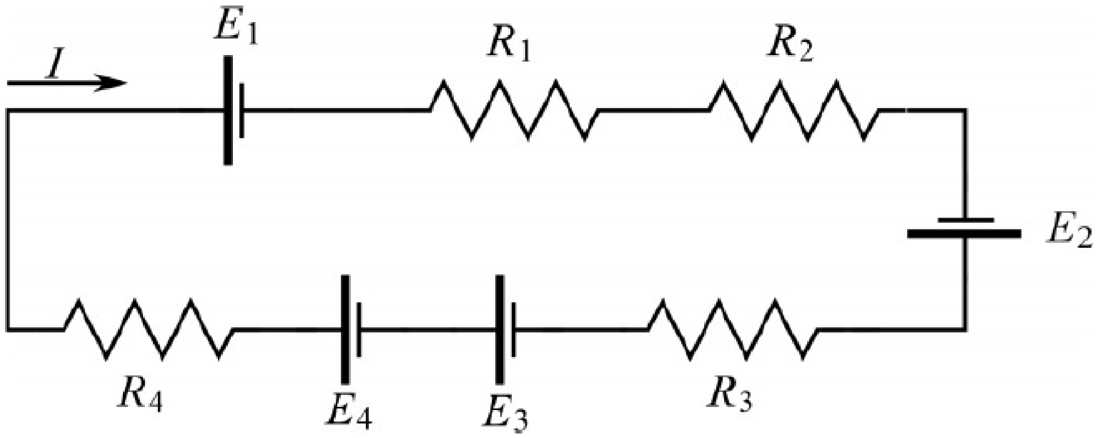
$$P_E = -EI$$

The circuit makes the current to flow in opposite direction in which the source, *E*, is pushing: In this case the source is subtracting energy from the circuit

Power balance

- In a circuit the sum of the power delivered = sum of the power dissipated:

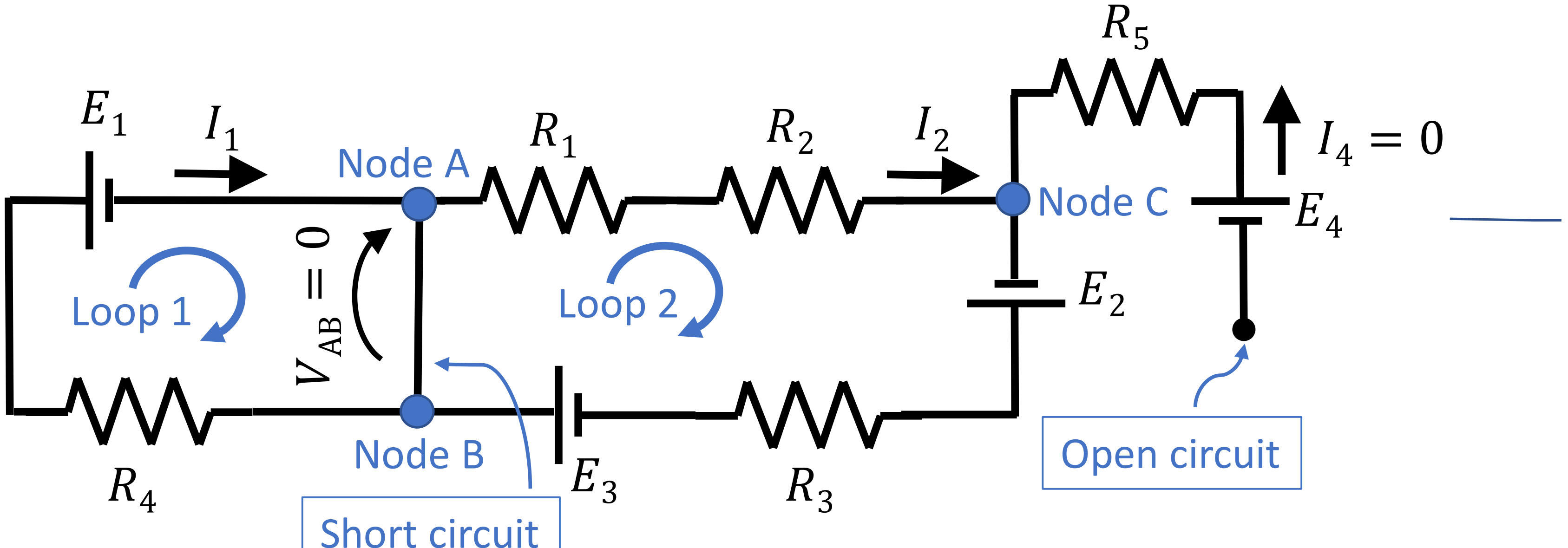
$$\sum_{n=1}^{N} P_{En} = \sum_{m=1}^{M} P_{Rm}$$



$$-E_1 I + E_2 I + E_3 I + E_4 I = R_1 I^2 + R_2 I^2 + R_3 I^2 + R_4 I^2$$

Main structural parts of a circuit or network

- **Branch**: single wire with or without elements.
- **Node**: any point where 3 or more branches are connected together.
- **Loop**: any closed path in a circuit.
- **Open circuit** (o.c.) or open terminal: branch with one end not connected. The charges in this branch can not move (the current there is zero).
- **Short circuit** (s.c.): wire without any element in it that connects to points of the circuit. The charges can flow through it but they will not change their energy (voltage difference is zero).



Example of circuit with 5 branches, 3 nodes and 2 elemental loops

Kirchhoff's Laws

- To solve a circuit or network means the determination of all the currents and/or voltages within it.
  - Each branch has associated a single current.
  - The voltage difference between nodes will allow the obtention of the currents along the branches using Ohm's law.
- Kirchhoff's laws allow determining the currents and/or voltages of electrical circuits when knowing the values of the electrical components (the resistances, voltage sources, etc.).
- There are two Kirchhoff's laws, one based on the charge conservation and the other on the energy conservation: …

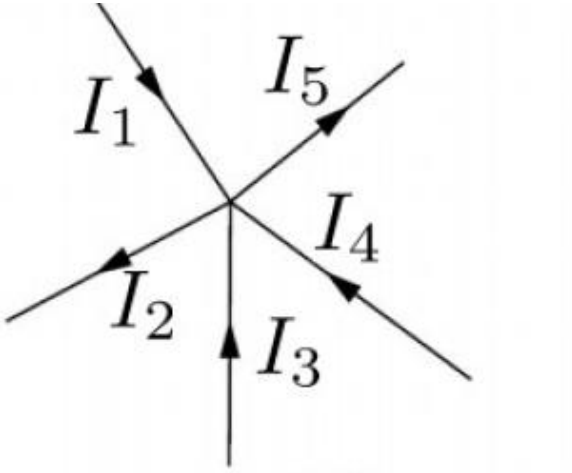
Kirchhoff's Laws

- **1st Law**: The sum of all the currents entering a node = sum of all the currents leaving it.

(This applies also to any part as well as to the whole of the circuit when connected to GND's)

$$\sum I_{entering} = \sum I_{leaving}$$

$$I_1 + I_3 + I_4 = I_2 + I_5$$



Kirchhoff's Laws

- **2nd Law:** The sum of all the voltage differences (calculated with the same direction) at each element along a closed loop is zero:

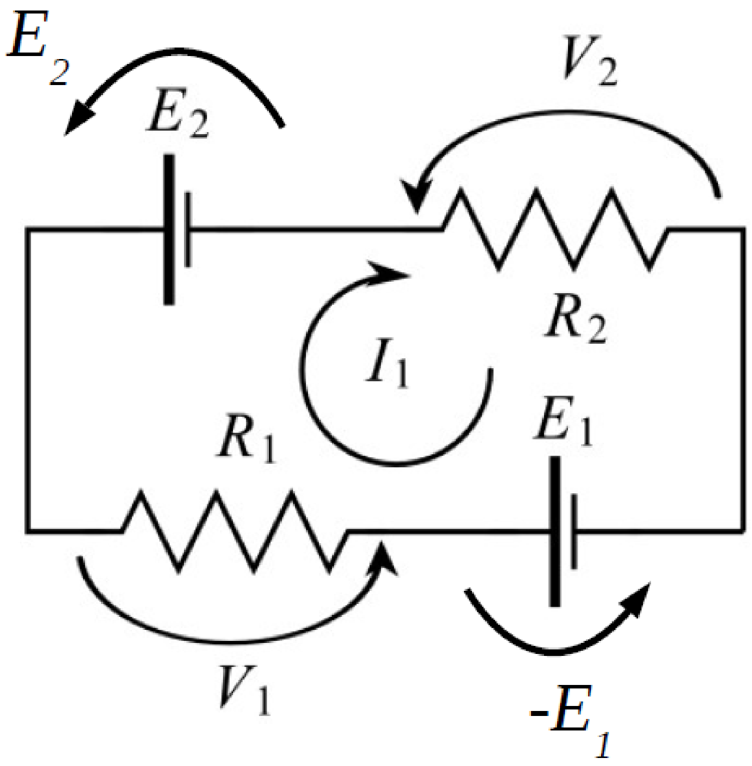
$$E_2 + V_2 - E_1 + V_1 = 0$$

$$\Downarrow$$

$$\underbrace{E_1 - E_2}_{\text{Supplied energy}} = \underbrace{R_1 I_1 + R_2 I_1}_{\text{dissipated energy}}$$

Supplied energy = dissipated energy

E_2 opposes to the current flow I_1



Kirchhoff's Laws

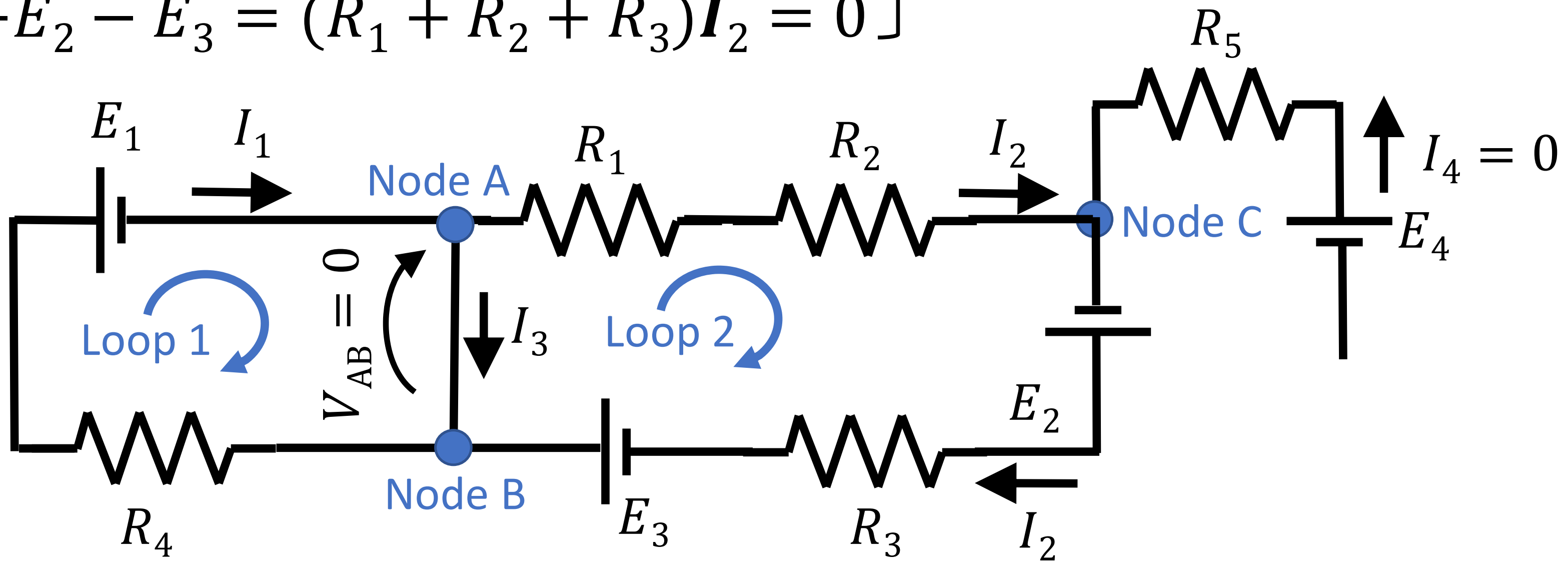
- Using Kirchhoff's Laws for solving the circuits:
  - Define the **Mesh currents*** for each *elemental loop* (smallest possible loops).
    - For loops with outside branches, the mesh currents coincide with the currents along the external branch (branches not sheared by the loops)
    - With the mesh currents (once calculated) the current along any internal branch can be calculated by applying the 1st Kirchhoff law (i.e. by subtracting them)
  - Apply the 2nd Kirchhoff law to each loop (without going over the current source if there were). If there are no current sources, there will be as many equations as unknown mesh currents (with current sources will be solved in module 3).

* All with the same clockwise or anticlockwise sense, for convenience.

Example of Kirchhoff law's application

- Mesh currents are I_1 and I_2
- 2nd Kirchhoff law:
  - Loop 1: $E_1 + R_4\boldsymbol{I}_1 = 0$
  - Loop 2: $-E_2 - E_3 = (R_1 + R_2 + R_3)\boldsymbol{I}_2 = 0$

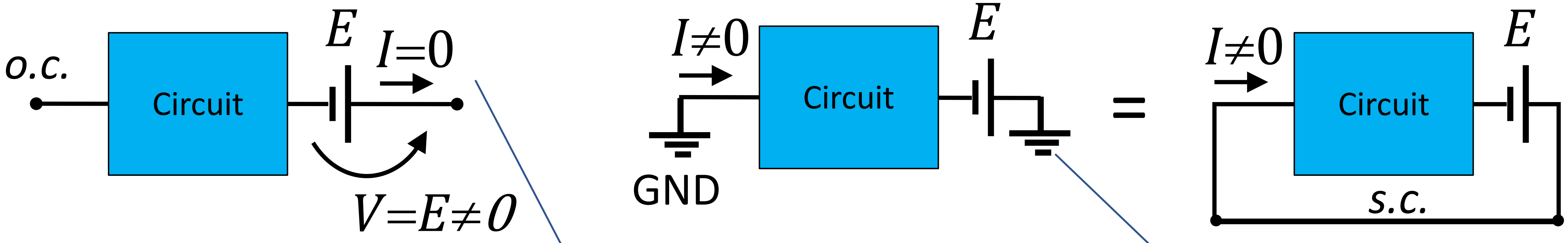
$\Rightarrow \{I_1 , I_2\}$



- From the calculated mesh currents using 1st Kirchhoff law: $I_3 = I_1 - I_2$

Important remarks

- There will be movement of charges (non zero current) only when the neutrality is fulfilled: If there is a current leaving part of the circuit, the same current has to enter this part of the circuit (1st Kirchhoff law).
- The ground connection allows the charges to leave or enter the circuit.
- In general the current flows from higher to lower voltage levels (always fulfilling the neutrality condition) except when a source is pushing the charges to a higher voltage level.
- There can be a huge voltage difference between two points A and B in a circuit but the charges will not move between A and B if they are not connected together with a loop (again fulfilling the neutrality condition)



I=0 because it is not fulfilling the neutrality condition due to open circuit even though E

I≠0 because the GND's allows the charge to move fulfilling the neutrality condition

Two GND is equivalent to make a short circuit between them

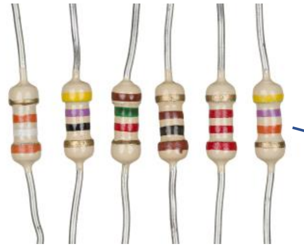
Electrical passive components

- The electrical *passive* components (they do not produce energy by their own) in linear electric circuits are
  - Resistors
  - Capacitors
  - Inductors
- These components will affect in different ways the movement of the charges when subjected to voltage differences. Their relation between voltage (applied at the terminals of the elements) and the current (through these elements) are different.

Resistors

- It is the electrical component characterized with just a value of *R*
- Energy dissipated by a resistor in DC after a time *T*

$$w_R(T) = VIT = RI^2T$$

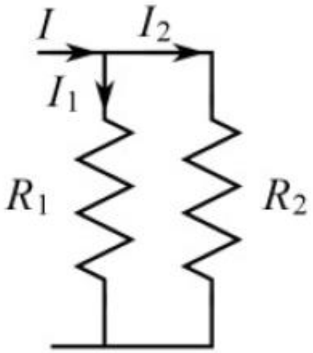


Color code for assigning *R* values

- According to Ohms law the relation between *v(t)* and *i(t)* in a resistor is trough the resistance *R* which is a real magnitude, therefore, there will be no *transient behavior* between them (i.e. there is no delay between *v(t)* and *i(t);* when one is changed the other follows instantaneously)

Current and voltage divider

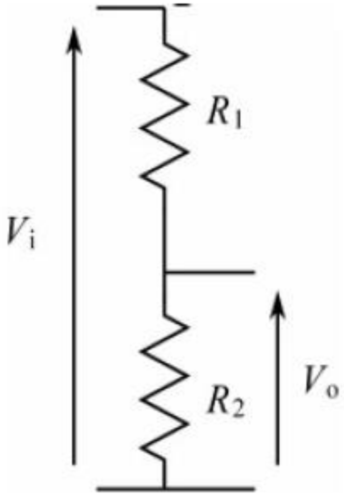
- Current divider



$$I_1 = I\frac{R_2}{R_1 + R_2}$$

$$I_2 = I\frac{R_1}{R_1 + R_2}$$

- Voltage divider

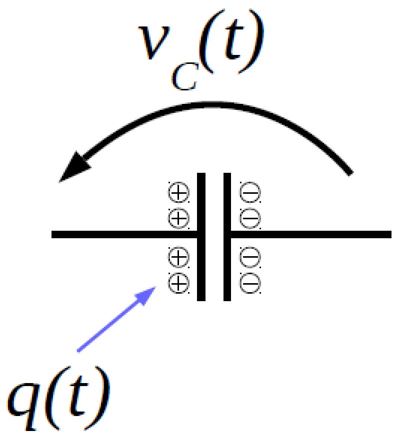


$$V_o = \frac{R_2}{R_1 + R_2}V_i$$

Capacitor

- It consist of two conducting plates separated by a dielectric material
- If a voltage is applied at its terminals the plates accumulates charge

$$q(t) = Cv_C(t)$$



The amount of accumulated charges for a given voltage is given by the **capacitance**, C, of the capacitor, with units Farads (F)

For a constant applied voltage difference (**DC**), once charged, there is no more movement of the charges accumulated on the plates and the current will be zero ($\Rightarrow$ **o.c.**) although the voltage is not zero!!

Capacitor

- The accumulated charges in the capacitor (positive on one plate and negatives on the other) implies an energy storage which is

$$\boxed{w_C(t) = \frac{1}{2C}q^2(t) = \frac{1}{2}Cv_C^2(t)}$$

The energy variation happens always in a continuous way, therefore the ***capacitors voltage will*** also ***change continuously***, and is used as initial condition when solving circuits.

- The current through *C* is zero for DC, since

$$\boxed{i_C(t) = \frac{\mathrm{d}q(t)}{\mathrm{d}t} = C\frac{\mathrm{d}v_C(t)}{\mathrm{d}t}}$$

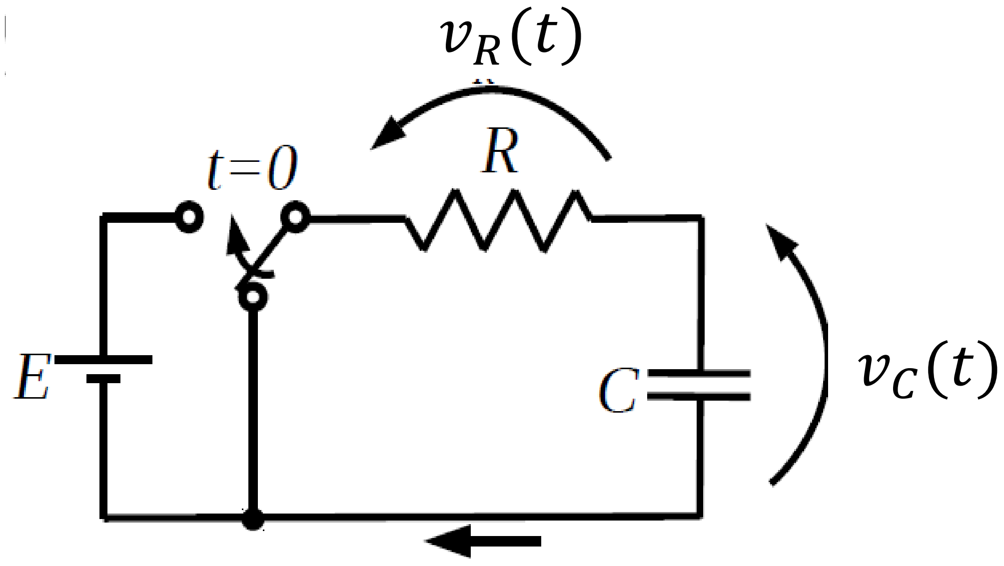
Capacitors transient behavior

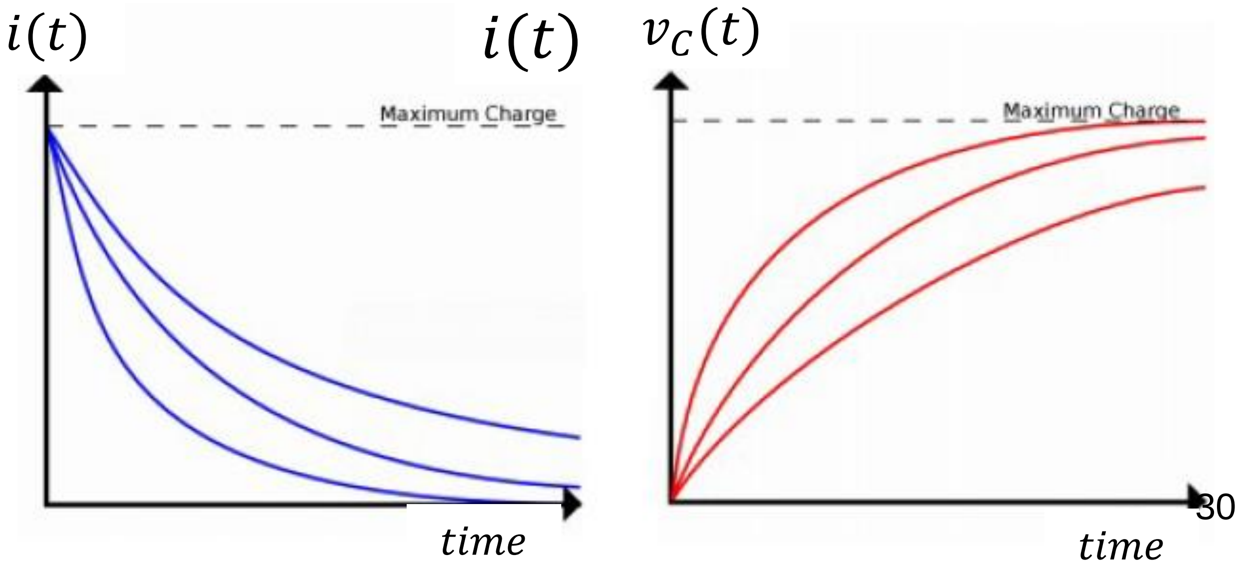
- For t>0 $\quad v_R(t) + v_C(t) = E$

$$Ri(t) + \frac{1}{C}q(t) = E$$

$$\frac{di(t)}{dt} + \frac{1}{RC}i(t) = 0$$

$$v_C(0) = 0 \Rightarrow \begin{cases} i(t) = \dfrac{E}{R}\mathrm{e}^{-\frac{t}{RC}} \\ v_C(t) = E(1 - \mathrm{e}^{-\frac{t}{RC}}) \end{cases}$$

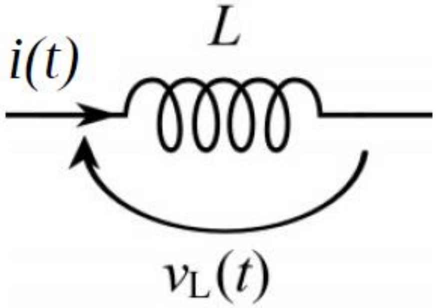


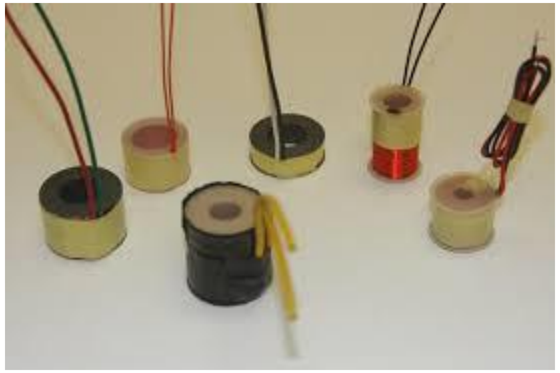


Inductor

- It is a coil of conducting wire.
- According to Ampere and Faraday, a varying current trough the coil induces a voltage difference at the coils terminal which is:

$$\boxed{v_L(t) = L\frac{di(t)}{dt}}$$



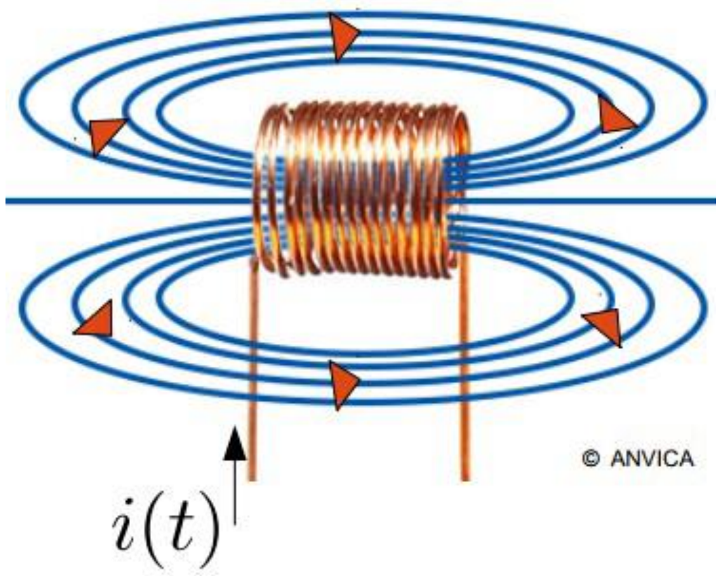


where *L* is the **inductance** with unit in *Henry's* (H).
If the current is constant (**DC**), the voltage is zero $\Rightarrow$ **s.c.** !!.

Inductor

- Also the inductor stores energy (in form of magnetic field) when a current is circulating through it. This energy at the instant *t* is:

$$\boxed{w_L(t) = \frac{1}{2} L i_L^2(t)}$$



The energy variation happens always in a continuous way, therefore the ***current trough a inductor will*** also ***change continuously***, and is used as initial condition when solving circuits. (The same happens for the capacitors voltage but with $v_C(t)$).

Inductors transient behavior

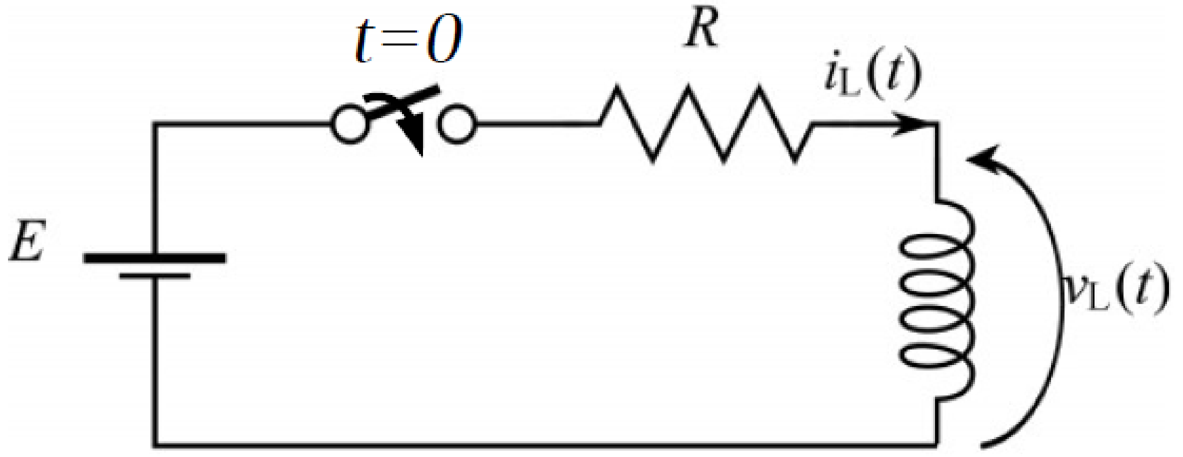
- For t>0 $$v_R(t) + v_L(t) = 0$$

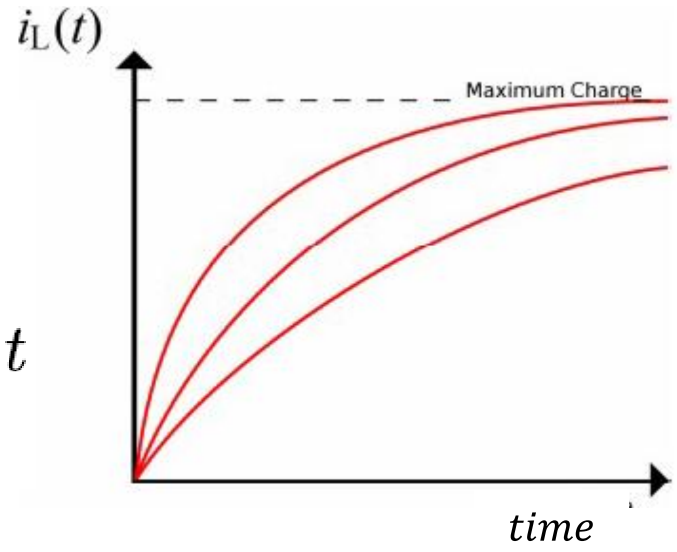
$$Ri(t) + L\frac{di(t)}{dt} = 0$$

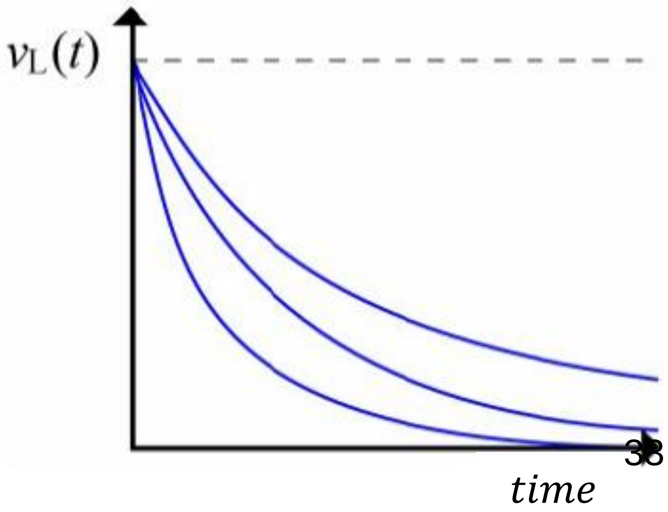
$$\frac{di(t)}{dt} + \frac{L}{R}i(t) = 0$$

$$i(0) = \frac{E}{R}$$

$$\begin{cases} i(t) = \dfrac{E}{R}\mathrm{e}^{-\frac{L}{R}t} \\ v_L(t) = -E\mathrm{e}^{-\frac{L}{R}t} \end{cases}$$





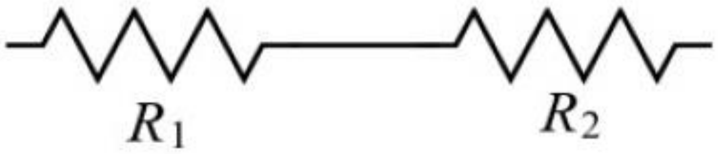


Remember

CAPACITOR	INDUCTOR
$i_C(t) = \frac{\mathrm{d}q(t)}{\mathrm{d}t} = C\frac{\mathrm{d}v_C(t)}{\mathrm{d}t}$ $W_C = \frac{1}{2}Cv_C^2$ **A capacitor is an open circuit to DC** 	$v_L(t) = L\frac{di_L(t)}{dt}$ $W_L = \frac{1}{2}Li_L^2$ **An inductor is a short circuit to DC** 

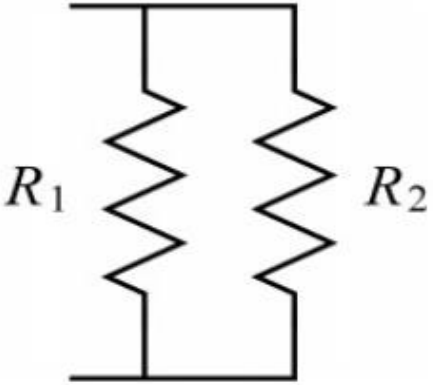
Association of resistors

- Associated in Series



$$R_{eq} = R_1 + R_2$$

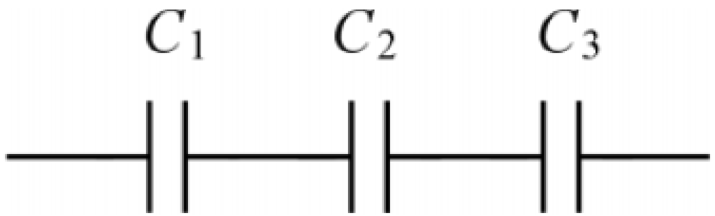
- Associated in parallel



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

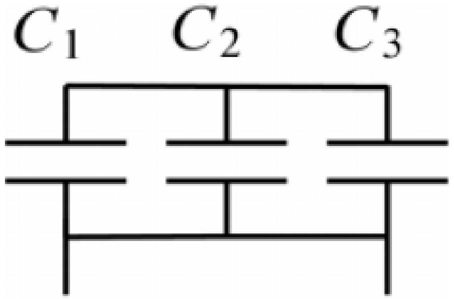
Association of capacitors

- Associated in Series



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

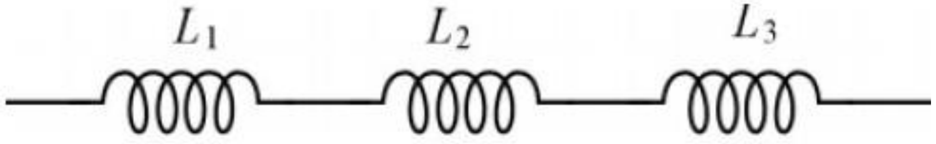
- Associated in parallel



$$C_{eq} = C_1 + C_2 + C_3$$

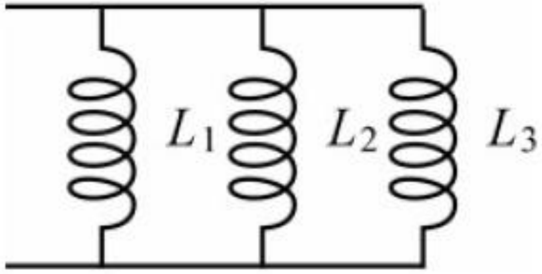
Association of inductors

- Associated in Series



$$L_{eq} = L_1 + L_2 + L_3$$

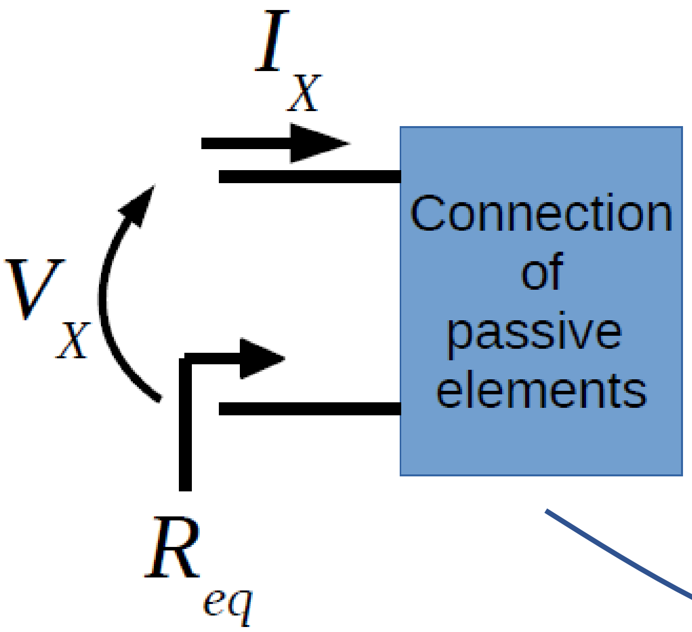
- Associated in parallel



$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

Equivalent resistance

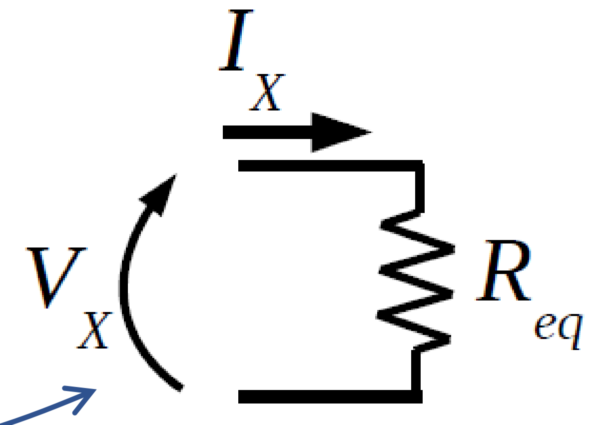
- Using Ohm's law, in DC it is possible to obtain an equivalent resistance between two terminals of several passive elements connected together



For any applied voltage there will be the corresponding current and their relation gives:

$$R_{eq} = \frac{V_X}{I_X}$$

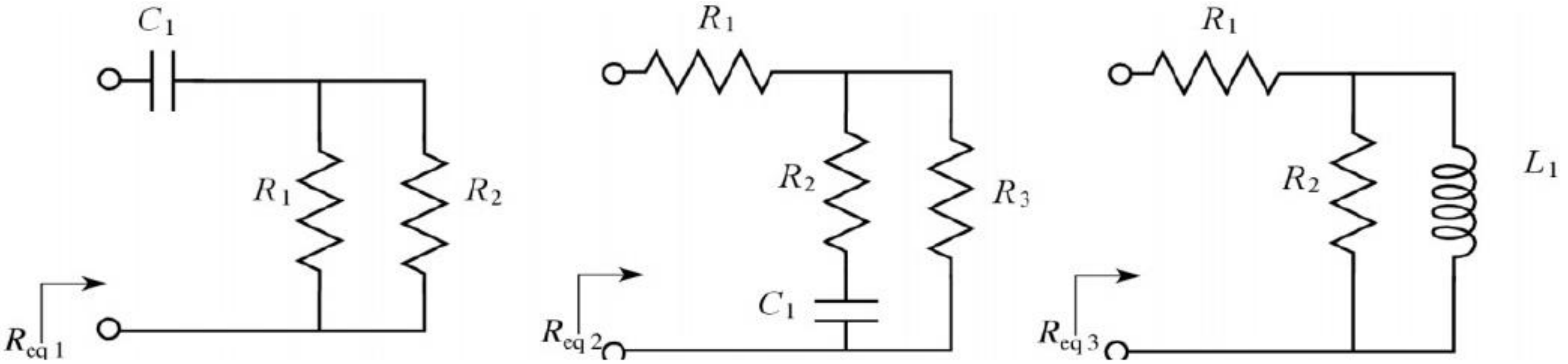
We can substitute the passive circuit by just a single equivalent Resistance:



Simplification with just one R_{eq}

Example of equivalent resistances

- Obtain the equivalent resistances of the following circuits if connected in DC



The inductor and the capacitor will have different electrical behaviors depending on the time evolution of the signal (if in DC or AC etc.) as we will see in the next module 2