EXERCISES MODULE 2: AC CIRCUITS SINUSOIDAL STEADY-STATE ANALYSIS

October 25, 2016

PROBLEM 2.1

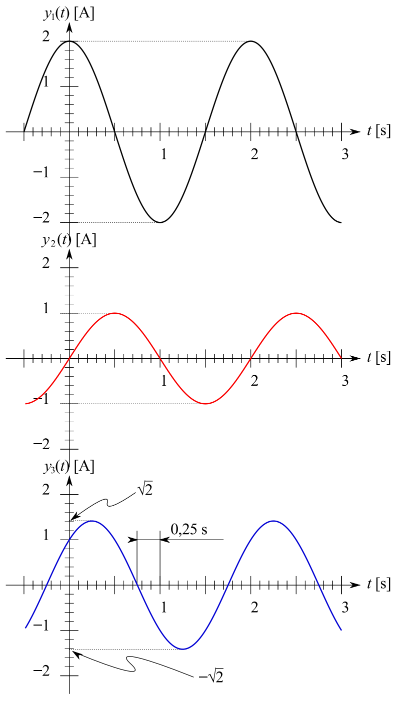


Figure 1

Given the signals in Figure 1, compute:

a) $x_1(t) = y_1(t) + y_2(t)$

b) $x_2(t) = y_1(t) + y_3(t)$

c) $x_3(t) = y_1(t) + y_2(t) - y_3(t)$

Result

1. $x_1(t) = \sqrt{5}\,\cos(\pi t - 0.463) = \sqrt{5}\,\sin(\pi t + 1.107)$
2. $x_2(t) = \sqrt{10}\,\cos(\pi t - 0.3217) = \sqrt{10}\,\sin(\pi t + 1.249)$
3. $x_3(t) = \cos(\pi t) = \sin\left(\pi t + \frac{\pi}{2}\right)$

PROBLEM 2.2

For the circuit in Figure 2, the signals $e(t)$ and $i(t)$ are shown in the graphs of Figure 3. Obtain the value of R and C.

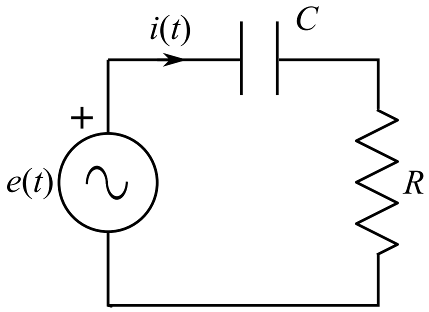


Figure 2

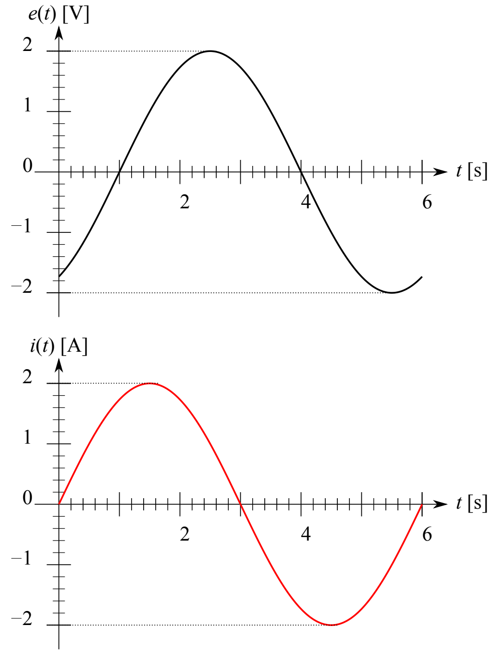


Figure 3

Result

$R = \frac{1}{2}\ \Omega$; $C = \frac{2\sqrt{3}}{\pi} = 1.103$ F

PROBLEM 2.3

For the circuit in Figure 4, currents $i_1(t)$ and $i_2(t)$ are shown in the graphs of Figure 5. If the power delivered by the source $e(t)$ is of 2 W, find the value of R_1, R_2 and L.

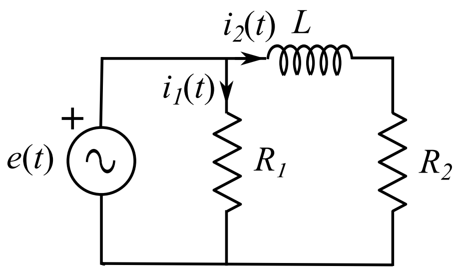


Figure 4

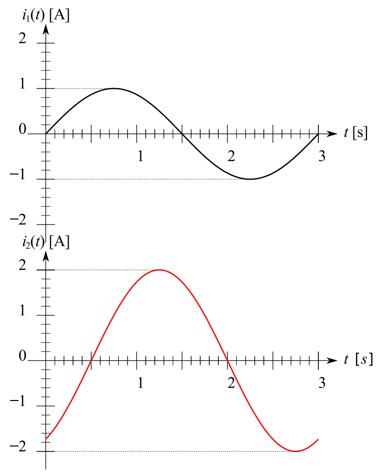


Figure 5

Result

$R_1 = 2\ \Omega; \quad R_2 = \dfrac{1}{2}\ \Omega; \quad L = \dfrac{3\sqrt{3}}{4\pi} = 0.413\ \text{H}$

PROBLEM 2.4

For the circuit in Figure 6, the graphs of Figure 7 represent the source $e(t)$ and the current $i_1(t)$. If $R_2 = 1\ \Omega$:

a) Compute the value of R_1 and C.

b) Determine the power of the source $e(t)$.

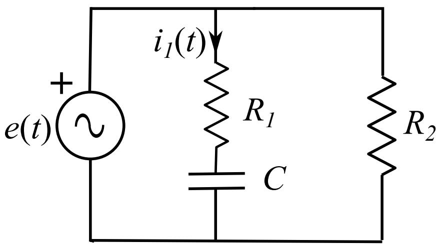


Figure 6

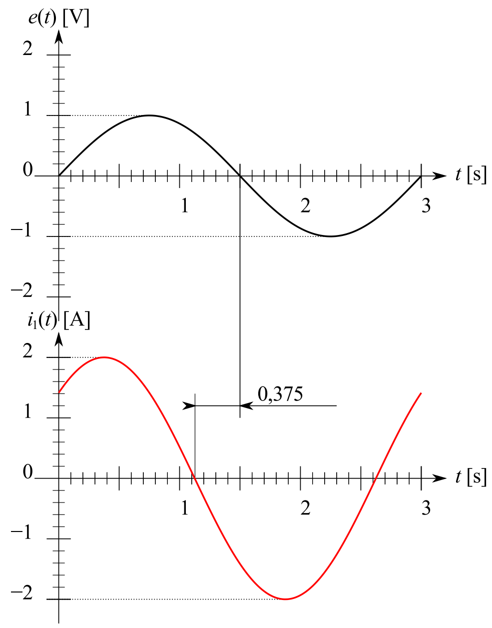


Figure 7

Result

a) $\begin{cases} R_1 = \dfrac{1}{2\sqrt{2}} = 0.354\ \Omega \\ C = \dfrac{3\sqrt{2}}{\pi} = 1.35\ \text{F} \end{cases}$

b) $P_{e(t)} = 1.207\ \text{W}$

PROBLEM 2.5

For the circuit in Figure 8, when $e(t)$ is substituted by the signal $e_1(t)$ shown in Figure 9 the current $i_C(t)$ is equal to 1 A. If $R = 1\ \Omega$, determine the difference of phase between the source and the current $i(t)$ when $e(t)$ follows the signal $e_2(t)$ shown in Figure 9.

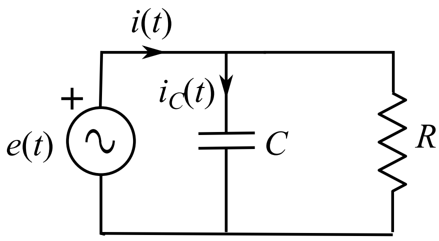


Figure 8

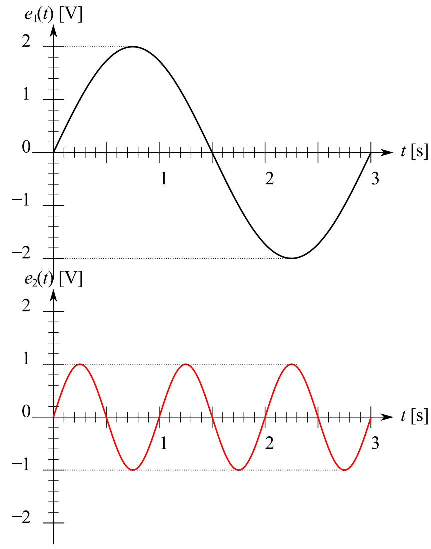


Figure 9

Result

$\varphi_{i(t)} - \varphi_{e(t)} = 0.983$ rad

PROBLEM 2.6

For the circuit in Figure 10 obtain the time-domain expressions for $i(t)$, $v_1(t)$ and $v_2(t)$.

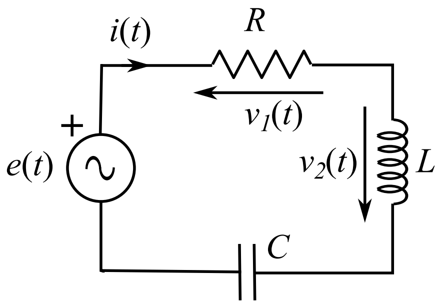


Figure 10

Data:

$$e(t) = \sqrt{2}\ \cos\left(10^6 t - \frac{\pi}{4}\right)\ \text{V};\quad L = 2\ \mu\text{H}$$
$$C = 1\ \mu\text{F};\qquad R = 1\ \Omega$$

Result

$$\begin{cases} i(t) = \cos\left(10^6 t - \dfrac{\pi}{2}\right)\ \text{A} \\ v_1(t) = \cos\left(10^6 t - \dfrac{\pi}{2}\right)\ \text{V} \\ v_2(t) = 2\ \cos\left(10^6 t + \pi\right)\ \text{V} \end{cases}$$

PROBLEM 2.7

For the circuit in Figure 11 obtain the time-domain expressions $v_C(t)$, $i_1(t)$, $i_2(t)$ and $i_3(t)$ and represent the phasor diagram of the circuit.

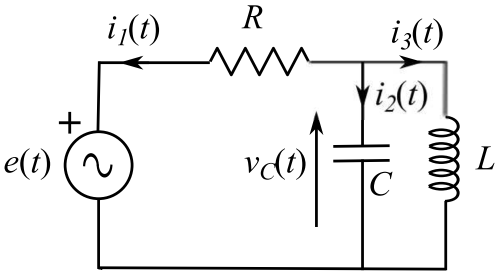


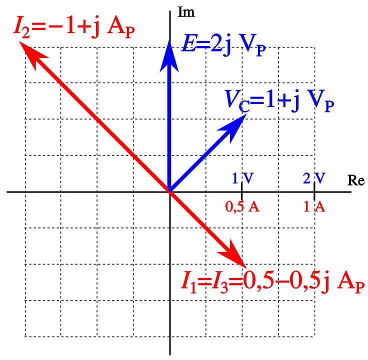
Figure 11

Data:

$$e(t) = 2\ \cos\left(10^6 t + \frac{\pi}{2}\right)\ \text{V};\quad L = 2\ \mu\text{H}$$
$$C = 1\ \mu\text{F};\quad R = 2\ \Omega$$

Result

$$\begin{cases} v_C(t) = \sqrt{2}\ \cos\left(10^6 t + \frac{\pi}{4}\right)\ \text{V} \\ i_1(t) = \frac{\sqrt{2}}{2}\ \cos\left(10^6 t - \frac{\pi}{4}\right)\ \text{A} \\ i_2(t) = \sqrt{2}\ \cos\left(10^6 t + \frac{3\pi}{4}\right)\ \text{A} \\ i_3(t) = \frac{\sqrt{2}}{2}\ \cos\left(10^6 t - \frac{\pi}{4}\right)\ \text{A} \end{cases}$$



PROBLEM 2.8

For the circuit in Figure 12 obtain;

a) The time-domain expressions for $i_1(t)$, $i_2(t)$, $i_3(t)$ and $i_4(t)$.

b) Power of the source $e(t)$.

c) Power absorbed by all the passive components of the circuit.

d) Power balance.

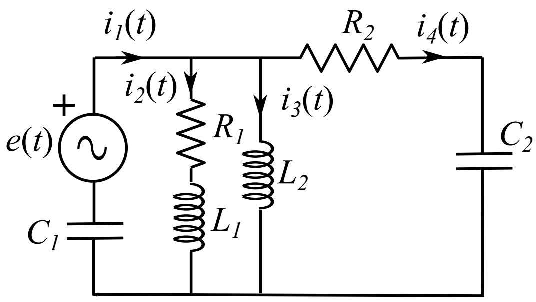


Figure 12

Data:

$$e(t) = \sqrt{2}\ \cos\left(10^6 t + \frac{\pi}{4}\right)\ \text{V};\quad L_1 = 1\ \mu\text{H}$$
$$L_2 = 1\ \mu\text{H};\quad C_1 = 2\ \mu\text{F}$$
$$C_2 = 1\ \mu\text{F},\quad R_1 = R_2 = 1\ \Omega$$

Result

a) $\begin{cases} i_1(t) = 2\sqrt{2}\ \cos\left(10^6 t + \dfrac{\pi}{4}\right)\ \text{A} \\ i_2(t) = \sqrt{2}\ \cos\left(10^6 t + \dfrac{\pi}{4}\right)\ \text{A} \\ i_3(t) = 2\ \cos\left(10^6 t\right)\ \text{A} \\ i_4(t) = \sqrt{2}\ \cos\left(10^6 t + \dfrac{3\pi}{4}\right)\ \text{A} \end{cases}$

b) $P_{e(t)} = 2$ W.

c) $P_{R_1} = P_{R_2} = 1$ W.

d) $P_{e(t)} = P_{R_1} + P_{R_2} = 2$ W.

PROBLEM 2.9

In the circuit of Figure 13 obtain the modulus of the impedance of the inductor $|Z_L|$.

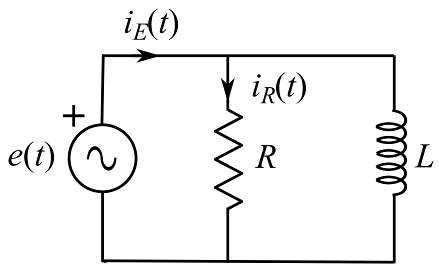


Figure 13

Data:

$$|I_R| = 1 \text{ A}$$
$$|I_E| = \sqrt{2} \text{ A}$$
$$R = 1\ \Omega$$

Result
$|Z_L| = 1\ \Omega$

PROBLEM 2.10

In the circuit of Figure 14 it is known that the source is delivering a power $P_{e(t)} = 2$ W, and that the current $i_L(t)$ **lags** the voltage $e(t)$. Determine:

a) Power of R_1.

b) Value of R_2.

c) Value of the modulus of the impedance of the capacitor $|Z_C|$.

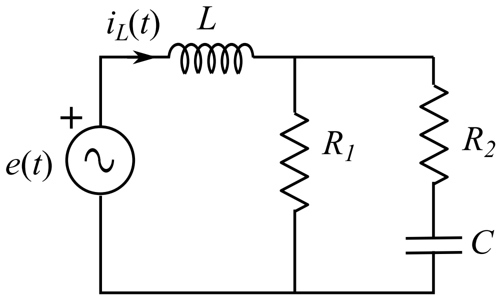


Figure 14

Data:

$$|E| = 2 \text{ V}$$
$$|I_L| = 2 \text{ A}_{ef}$$
$$R_1 = 2\ \Omega$$
$$|Z_L| = 1\ \Omega$$

Result

a) $P_{R_1} = 1$ W ; b) $R_2 = \frac{2}{5}\ \Omega$; c) $|Z_C| = \frac{4}{5}\ \Omega$.

PROBLEM 2.11

For the circuit in Figure 15, voltage $v_{CD}(t)$ and current $i_R(t)$ are shown in Figure 16. Determine:

a) Values of R and C.

b) If $|I_E| = 1$ A_{ef}, the value of L

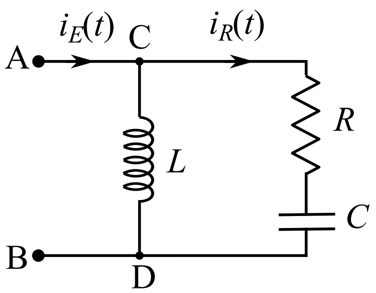


Figure 15

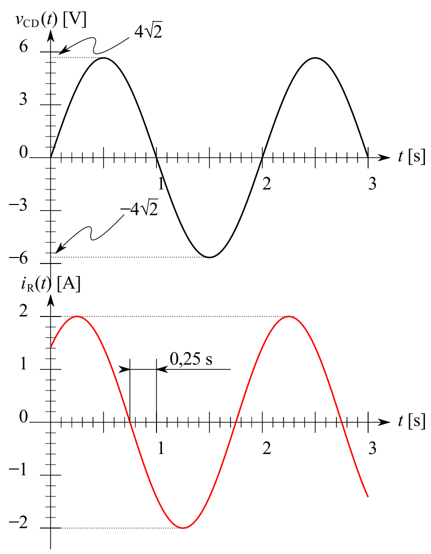


Figure 16

Result

a) $R = 2\ \Omega; \quad C = \dfrac{1}{2\pi} = 0.159$ F.

b) $L = \dfrac{4}{\pi} = 1.273$ H.

PROBLEM 2.12

For the circuit in Figure 17 determine the value of $|Z_C|$.

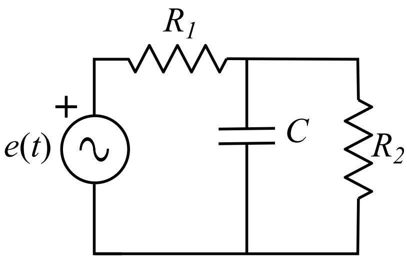


Figure 17

Data:

$$P_{R_2} = 9\ \mathrm{W}$$
$$P_{e(t)} = 34\ \mathrm{W}$$
$$R_1 = R_2 = 2\ \Omega$$

Result
$|Z_C| = \dfrac{3}{2}\ \Omega.$

PROBLEM 2.13

For the circuit in Figure 18, obtain the expressions for V_1, V_2 and V_3 as a function of I_1, I_2, ωL_1, ωL_2, ωM_{12}, ωM_{13} and ωM_{23}.

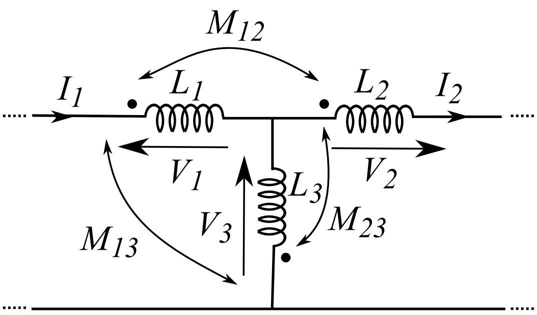


Figure 18

Result

$$\begin{cases} V_1 = j\omega\,(L_1 - M_{13}) \cdot I_1 + j\omega\,(M_{12} + M_{13}) \cdot I_2 \\ V_2 = j\omega\,(M_{23-M_{12}}) \cdot I_1 - j\omega\,(L_2 + M_{23}) \cdot I_2 \\ V_3 = j\omega\,(L_3 - M_{13}) \cdot I_1 - j\omega\,(L_3 + M_{23}) \cdot I_2 \end{cases}$$

PROBLEM 2.14

For the circuit in Figure 19 obtain the time-domain expressions for $v_1(t)$, $v_2(t)$, and $v_3(t)$.

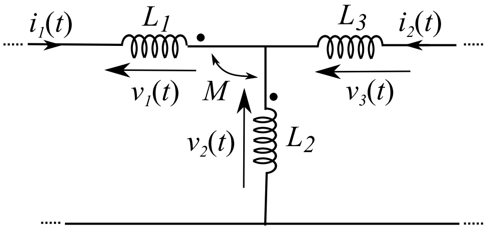


Figure 19

$$Data: \quad i_1(t) = sen\,(\omega t)\ A \quad ; \quad i_2(t) = sen\left(\omega t + \frac{\pi}{2}\right)\ A$$
$$\omega L_1 = \omega L_2 = \omega L_3 = 2\ \Omega \quad ; \quad k = \frac{1}{2}$$

Result

$$\begin{cases} v_1(t) = \sqrt{2} \cdot sen\left(\omega t + \frac{\pi}{4}\right)\ V \\ v_2(t) = \sqrt{5} \cdot sen\,(\omega t + 2.678)\ V \\ v_3(t) = 2 \cdot sen\,(\omega t)\ V \end{cases}$$

PROBLEM 2.15

For the circuit in Figure 20, compute the time-domain expression of the electric potential difference between terminals A and B.

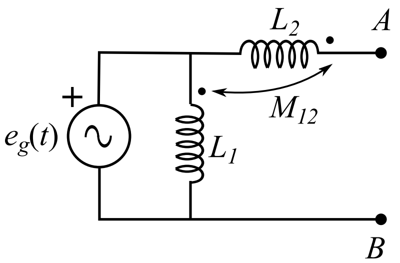


Figure 20

Data: $e_g(t) = 2 \cdot \cos(\omega t)$ V ; $\omega L_1 = \omega L_2 = 2\ \Omega$; $k = 0.5$

Result

$v_{AB}(t) = 3 \cdot \cos(\omega t)$ V

PROBLEM 2.16

For the circuit in Figure 21, obtain the relation $Z_{eq} = \dfrac{E}{I_1}$

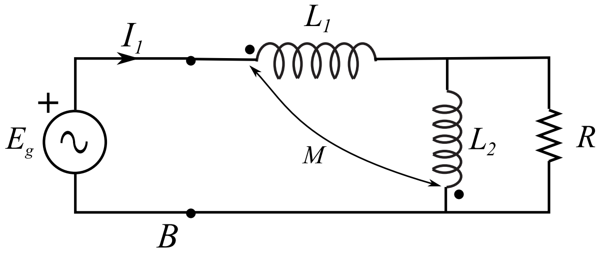


Figure 21

Data : $e_g(t) = E_0 \sin\left(10^3 t\right)\ V;\quad R = 2\ \Omega$

$L_1 = 1\text{mH};\quad L_2 = 2\text{mH};\quad k = \dfrac{1}{\sqrt{2}}$

Result

$Z_{eq} = \dfrac{1+3j}{4}\ \Omega$