Module 3. Sources, Resolution methods and equivalent impedances and sources

Circuit Theory, GIT
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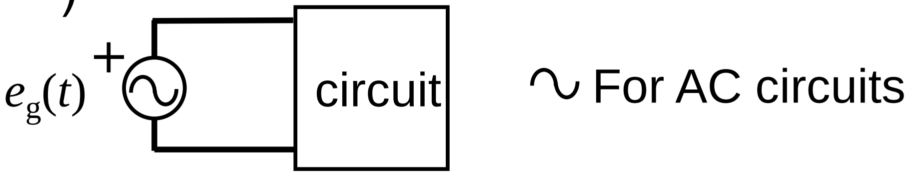
Sources, Resolution methods and equivalent impedances and sources

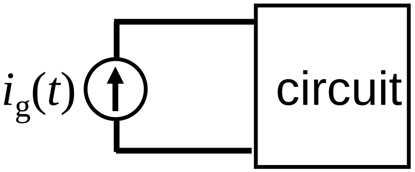
- Sources
  - Ideal sources
  - Real sources
  - Association of sources
- Systematic circuit analysis
  - Mesh analysis
  - Nodal analysis
- Equivalent impedance
- Equivalent Thevenin and Norton

Sources, recall

- Sources are the circuit elements that provides the energy to the charges.
  - **Voltage sources**: They provide a given voltage, $e_g(t)$, to the positive charges (q+) that passes trough it in a direction at the instant t=0 (indicated with "+")

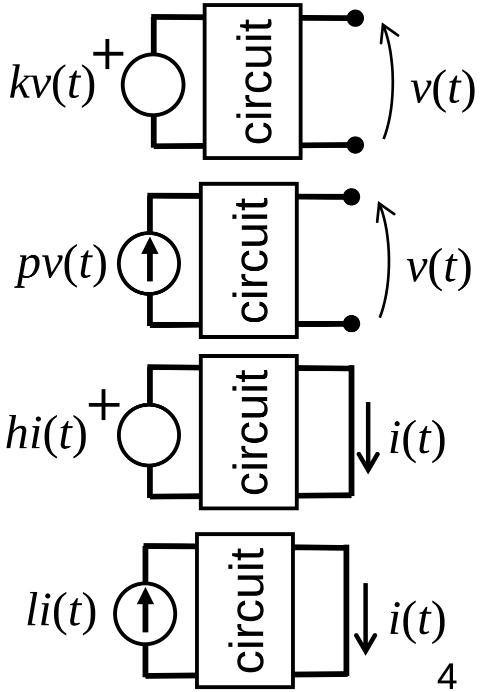


  - **Current sources:** They produce a given current, $i_g(t)$, in a given direction (indicated with "→") by providing the necessary voltage to the q+



Dependent Sources

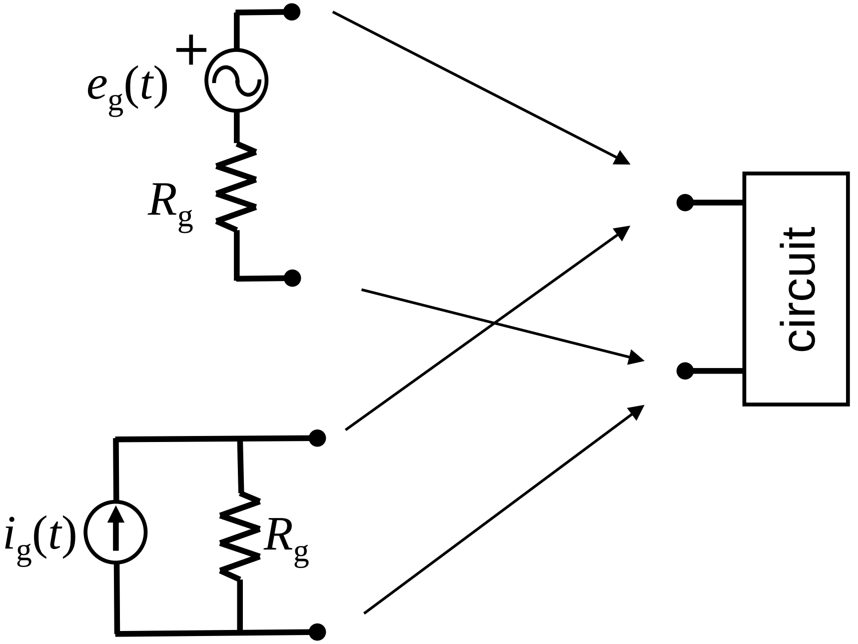
- Dependet sources are current or voltage sources whose value is the voltage or current somewhere else in the circuit multiplied by a constant factor.
- They are used for example for modeling the behavior of amplifiers.
- They can be
  - Voltage-controlled ***voltage*** source:
  - Current-controlled ***voltage*** source:
  - Voltage-controlled ***current*** source:
  - Current-controlled ***current*** source:



Units of p [=] Ω^{-1}, units of h [=] Ω

Real sources

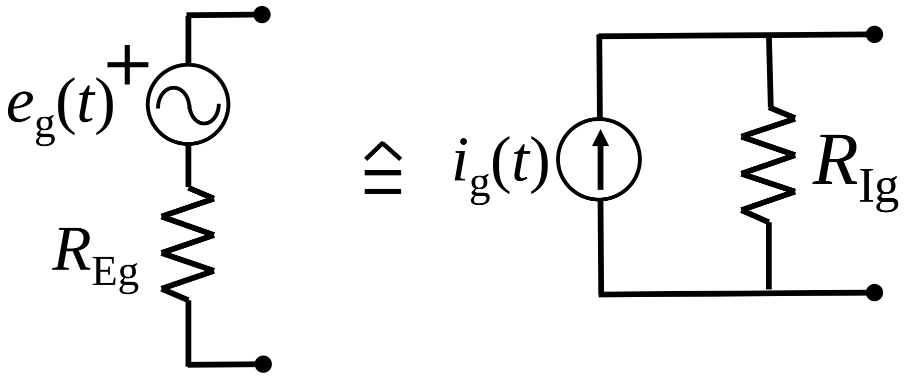
- The real sources have internal resistances, R_g, that accounts for their real behavior.
- Real voltage source
- Real current source



Equivalence between real sources

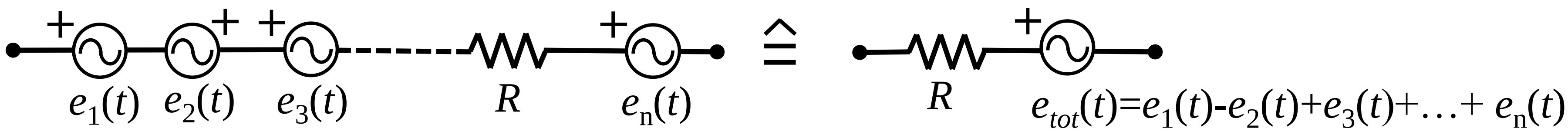
- The real current and voltage sources can be replaced one for the other if they are equivalent. They will produce the same effect on the circuit.
- They are equivalent when

$$\begin{cases} e_g(t) = R_{Ig} i_g(t) \\ R_{Eg} = R_{Ig} \end{cases}$$

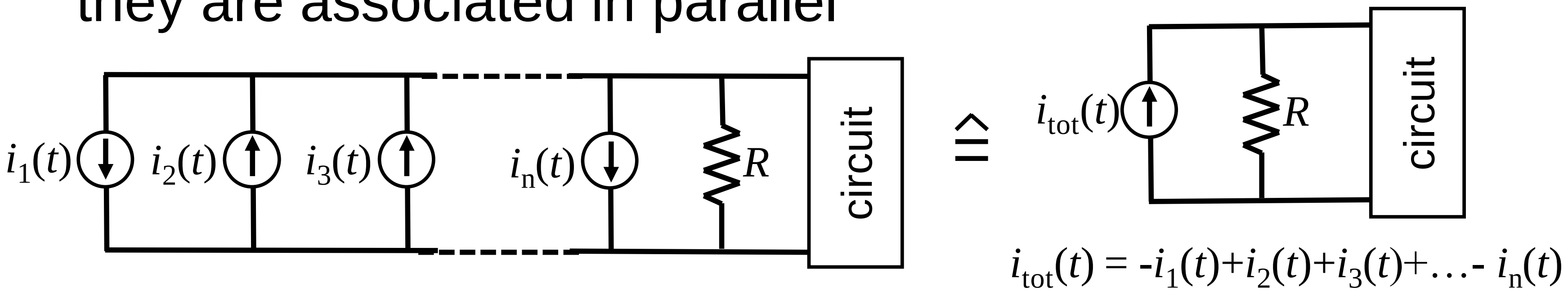


Association of ideal sources

- Ideal voltage sources can be associated only if they are connected in serial



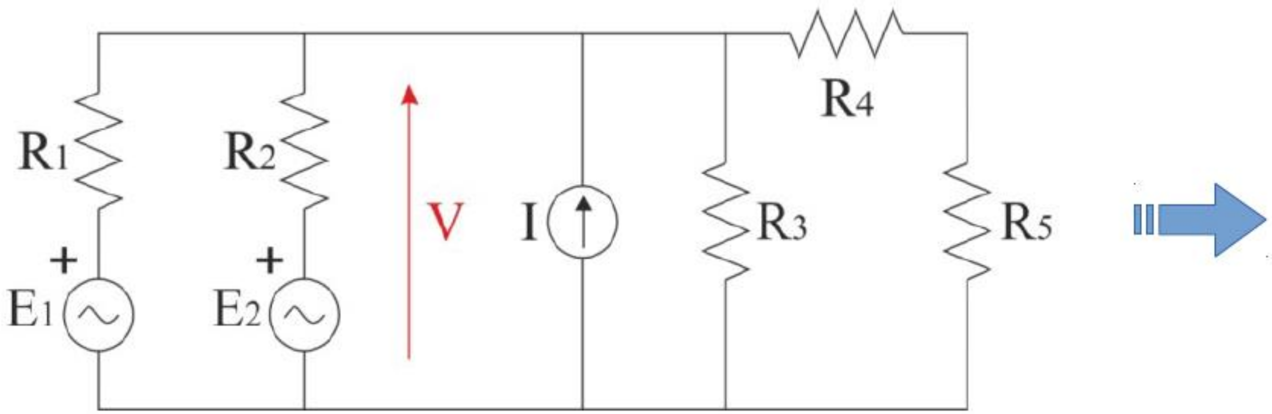
- Ideal current sources can only be associated in serial if they are associated in parallel



- Both real sources can be associated and simplified with one equivalent real source…

Association of real sources

- As example the following circuit is simplified



$$E_1 = 8V.; E_2 = 16V.; I = 10mA.; R_1 = 4k\Omega; R_2 = 4k\Omega; R_3 = 4k\Omega; R_4 = 2k\Omega; R_5 = 2k\Omega$$

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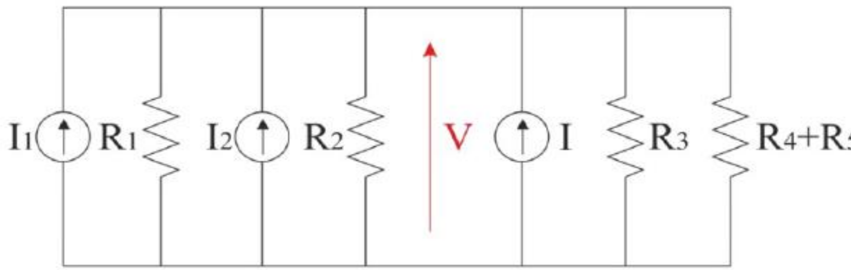
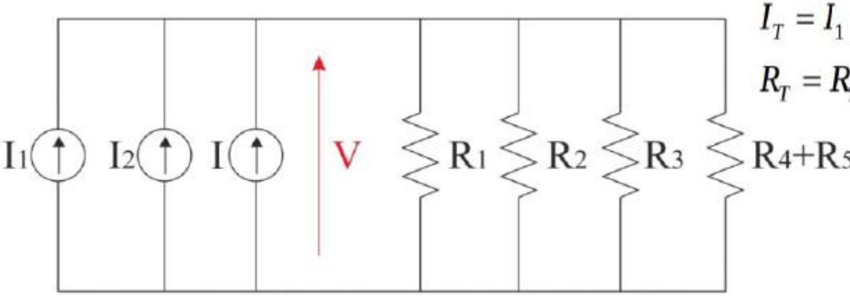
Association of real sources

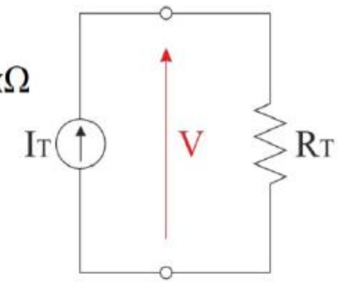
$$I_1 = \frac{E_1}{R_1} \quad ; \quad I_2 = \frac{E_2}{R_2}$$





$$I_T = I_1 + I_2 + I = 16mA$$

$$R_T = R_1 // R_2 // R_3 // (R_4 + R_5) = 1k\Omega$$



$$V = I_T \cdot R_T = 16 \cdot 10^{-3} \cdot 10^{3} = 16V.$$

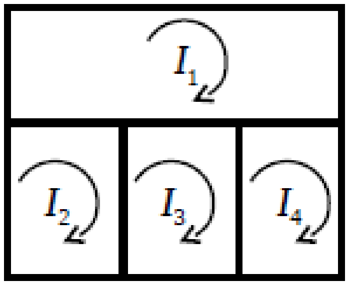
Systematic circuit analysis method

- To solve a circuit means to determine the values of the currents or voltages from which all the rest of the currents and voltages in the circuit can be obtained.
- Mesh analysis method uses the 2nd Kirchhoff law for determining the *mesh* currents.
- Nodal analysis method uses the 1st Kirchhoff law for determining the voltages at several nodes with respect to a reference node (i.e. GND)

Mesh analysis method

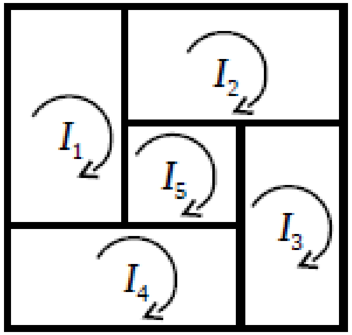
- It uses the 2nd Kirchhoff law (energy conservation)
- The circuit is solved when all the variables called **mesh currents** (I_1, I_2, …) are known.
- There is a mesh current for each smallest closed loop (i.e. **elemental mesh**) within the circuit:

Ex.1)



Variables are: I_1, I_2, I_3 and I_4
⇒ 4 linear independent equations

Ex.2)

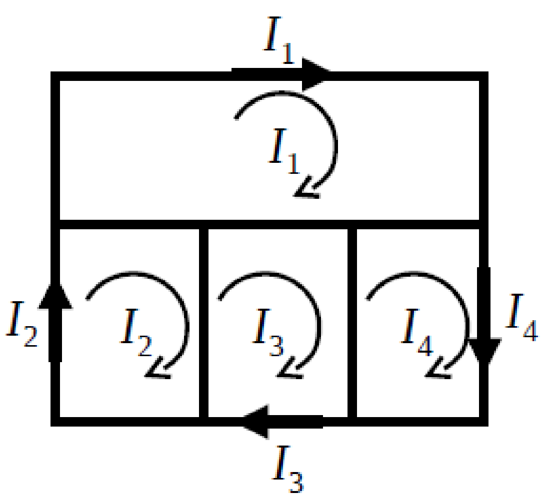


Variables are: I_1, I_2, I_3, I_4, and I_5
⇒ 5 linear independent equations

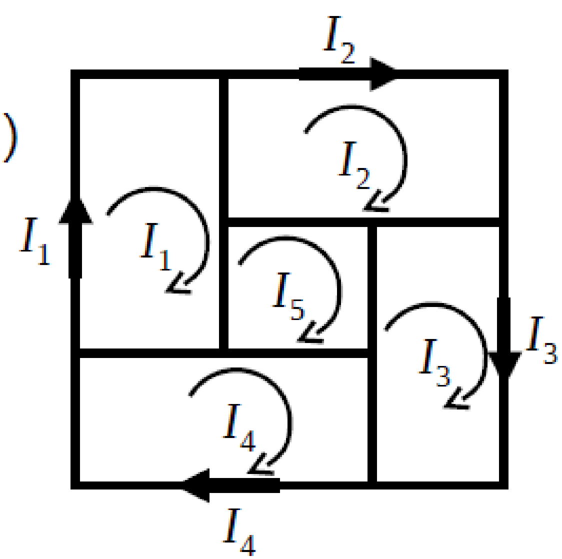
Mesh analysis method

- For convenience, each *mesh current* should have the same clockwise or anti-clockwise direction.
- The mesh currents are then:
  - Those that flows along the external wire of the elemental mesh (i.e. not sheared with another mesh)
  - If the mesh current is from a mesh that has no "external wire" (like I_5 in Ex. 2) then it does not really flow trough any wire

Ex.1)



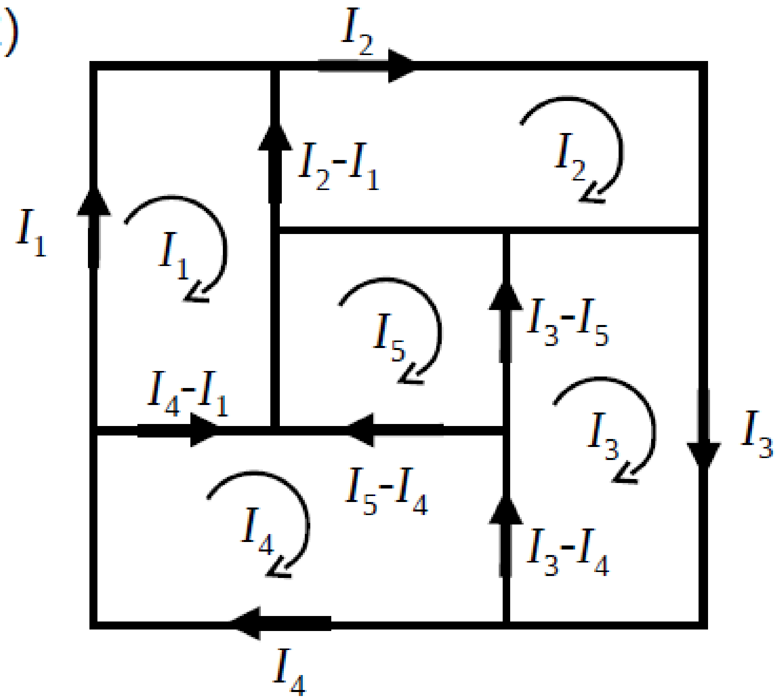
Ex.2)



Mesh analysis method

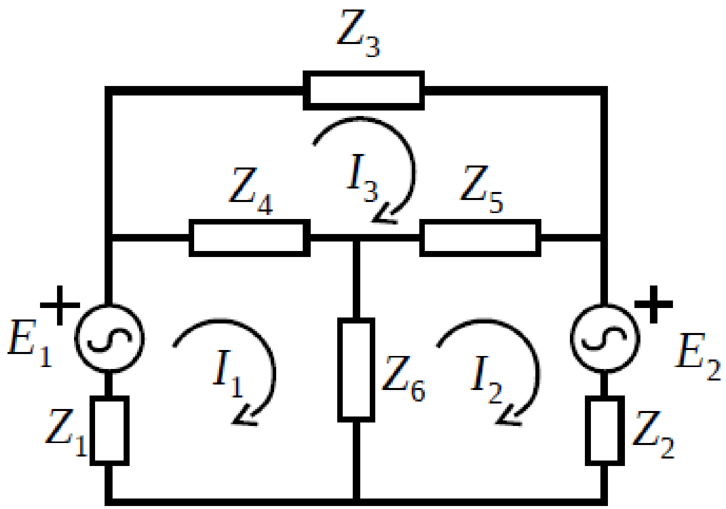
- From the mesh currents it is easy to get all the currents that flow along each wire of the circuit by just subtracting (or adding) the mesh currents belonging to the meshes on each side of the wire:

Ex.2)



Mesh analysis method

- Example 1

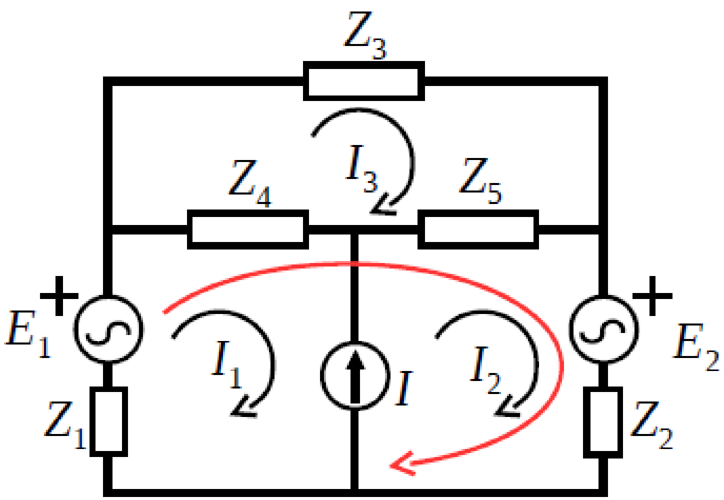


1. Identify the elemental meshes and assign the corresponding mesh currents (e.g. $\{I_1, I_2, I_3\}$). This will give you the number of unknown and the number of necessary independent equations to solve the circuit
2. If possible use 2nd Kirchhoff law for each mesh.

$$\left.\begin{aligned} E_1 &= Z_1 I_1 + Z_4(I_1 - I_3) + Z_6(I_1 - I_2) \\ -E_2 &= Z_2 I_2 + Z_6(I_2 - I_1) + Z_5(I_2 - I_3) \\ 0 &= Z_3 I_3 + Z_5(I_3 - I_2) + Z_4(I_3 - I_1) \end{aligned}\right\} \begin{pmatrix} E_1 \\ -E_2 \\ 0 \end{pmatrix} = \begin{pmatrix} Z_1 + Z_4 + Z_6 & -Z_6 & -Z_4 \\ -Z_6 & Z_2 + Z_6 + Z_5 & -Z_5 \\ -Z_4 & -Z_5 & Z_3 + Z_4 + Z_5 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix}$$

Mesh analysis method

- Example 2



$$\begin{cases} E_1 - E_2 = Z_1 I_1 + Z_4(I_1 - I_3) + Z_5(I_2 - I_3) + Z_2 I_2 \\ 0 = Z_3 I_3 + Z_5(I_3 - I_2) + Z_4(I_3 - I_1) \\ I = I_2 - I_1 \end{cases}$$

(*) you will lose a mesh equation

1. Identify the elemental meshes and assign the corresponding mesh currents $\{I_1, I_2, I_3\}$.
2. If possible use 2nd Kirchhoff law for each mesh.
3. If there exist a **current source** ($\boldsymbol{I}$)
   - Your **mesh equation should not go through the current source** (*).
   - Instead of the missing mesh equation you can use the equation:

     $I=I_2-I_1$

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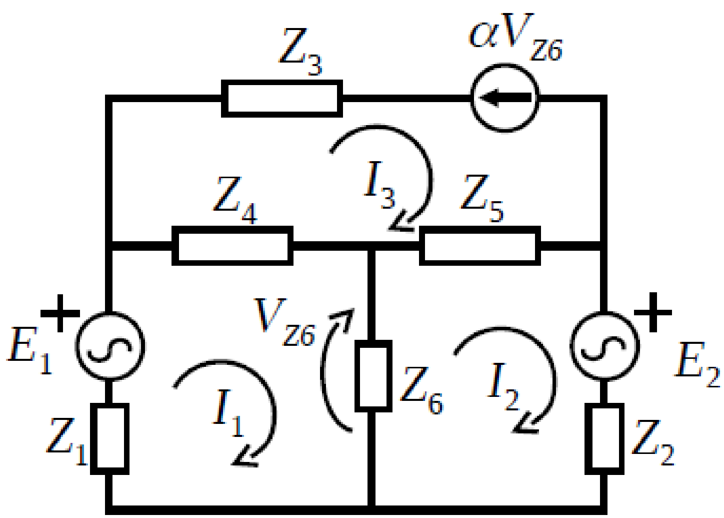
Mesh analysis method

- Example 3



$$\begin{cases} E_1 = Z_1 I_1 + Z_4(I_1 - I_3) + Z_6(I_1 - I_2) \\ -E_2 = Z_2 I_2 + Z_6(I_2 - I_1) + Z_5(I_2 - I_3) \\ \alpha V_{Z6} = -I_3 \\ V_{Z6} = Z_6(I_1 - I_2) \end{cases}$$

1. Identify the elemental meshes and assign the corresponding mesh currents
2. If possible use 2nd Kirchhoff law for each mesh
3. If there exist a current source (e.g. αV_{Z6}) instead of a mesh equation use:

   $$\boldsymbol{\alpha V_{Z6} = -I_3}.$$

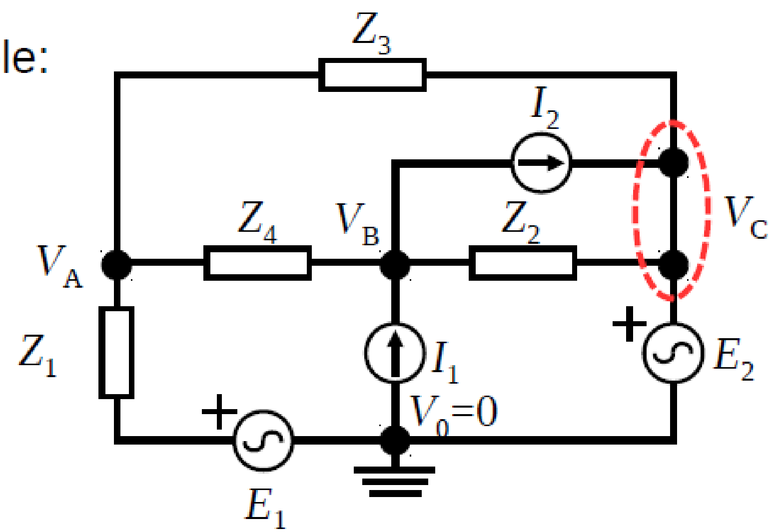
4. If there exist a **dependent source** then a new variable appears that has to be solved as a function of the mesh currents:

   $$\boldsymbol{V_{Z6} = Z_6\,(I_1 - I_2)}$$

Nodal analysis method

- It uses 1st Kirchhoff's law to obtain the equations (charge conservation)
- Variables are the nodal voltages (V_A, V_B, ...)
- The circuit is solved when the **nodal voltages** at all the nodes but one (i.e. the reference node) are known.

Example:



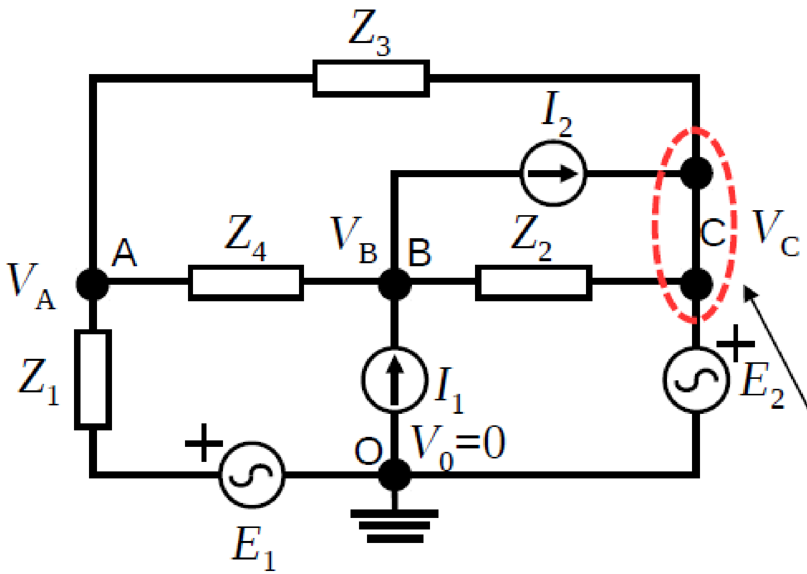
Variables are: V_A, V_B and V_C

The reference node is V_0=0 (the ground connection)

⇒ 3 linear independent equations

Nodal analysis method

Example 1:



Variables are: V_A, V_B and V_C

⇒ 3 linear independent equations

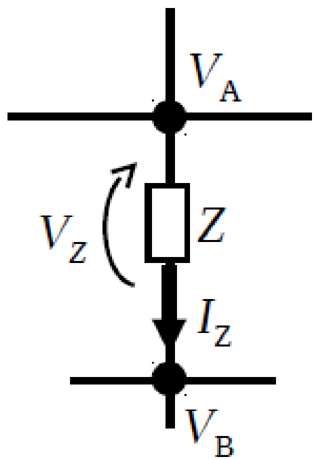
This accounts for one node

1. Identify all the nodes (A,B,C,…) for which the **nodal voltages** will be calculated. This will give you the "*N*" number of unknown. If there is no ground connection, choose any node as your reference node.
2. Apply the 1st Kirchhoff law to all the "*N*" different nodes you can.
3. If there exist a voltage source (e.g. E_2) without any other element on the same wire between two nodes then use the relation:

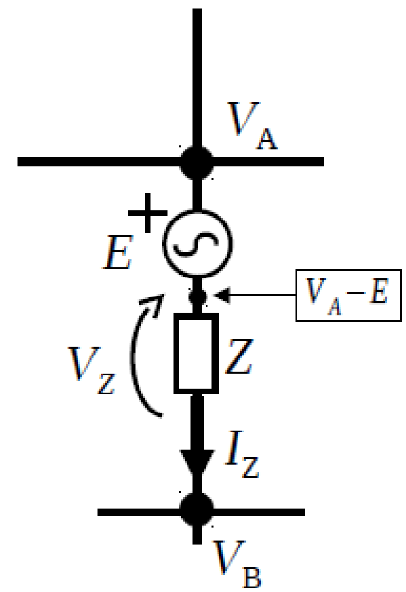
$$\boldsymbol{V_C - V_0 = E_2}$$

Nodal analysis method

- Examples for how to calculate the currents that flows trough a impedance as a function of the nodal voltages



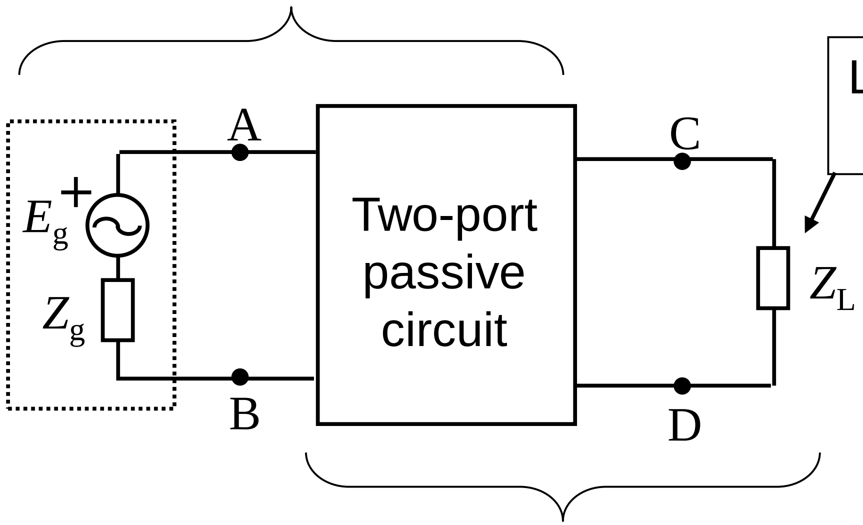
$$I_Z = \frac{V_Z}{Z} = \frac{V_A - V_B}{Z}$$



$$I_Z = \frac{V_Z}{Z} = \frac{(V_A - E) - V_B}{Z}$$

Schematic of simplified circuits

Active part of the circuit: Equivalent Thevenin, (E_{Th}, Z_{Th}), or Norton

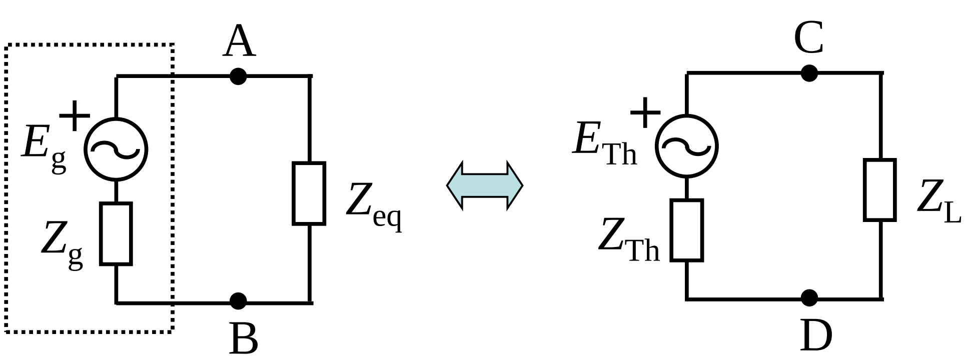


Load impedance (receiver)

Energy supply (real voltage or current source)

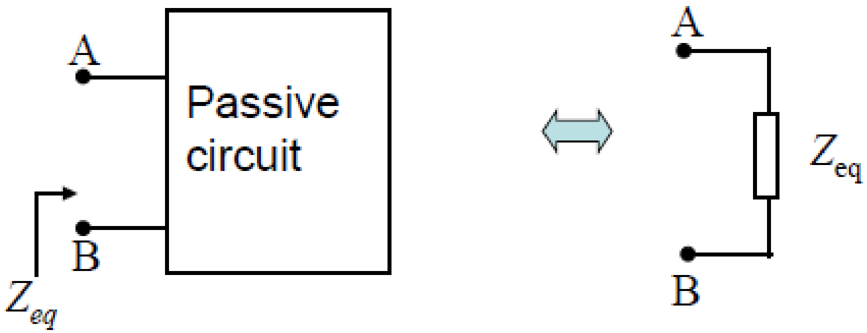
Passive part of the circuit: Equivalent impedance, Z_{eq}

Equivalent circuits:

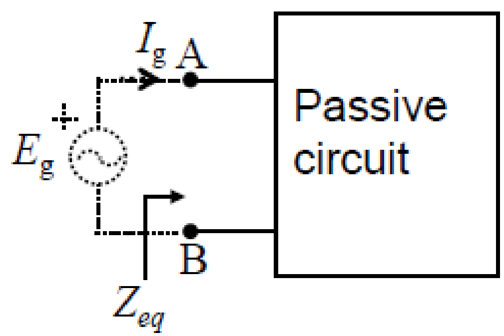


Equivalent impedance

- Any **passive** circuit (i.e. without independent sources) between two terminals (A,B) can be simplified with an equivalent impedance, Z_{eq}:



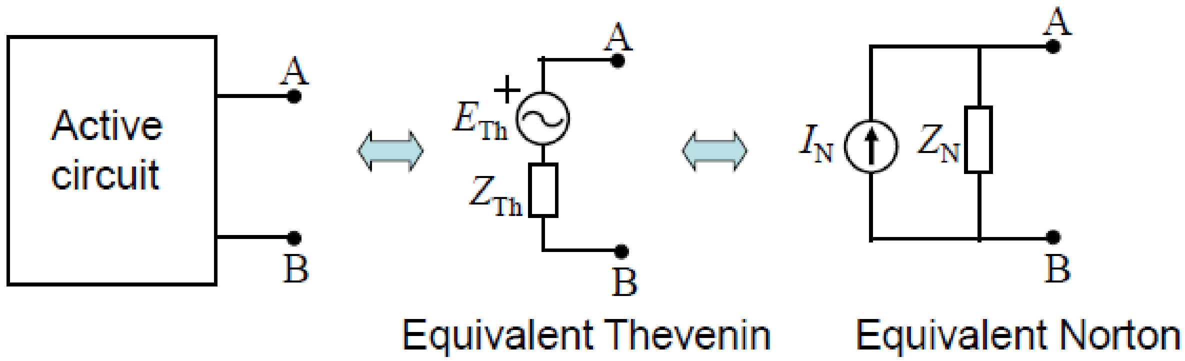
The equivalent impedance can be obtained by:
• Parallel and/or serial association of the impedances within the passive circuit.
• Applying a test generator E_g:



$$Z_{eq} = \frac{E_g}{I_g}$$

Equivalent active circuit

- Any linear circuit with independent sources (i.e. **active** circuit) between two terminals (A,B) can be simplified by a real voltage source (Equiv.Thevenin) or real current source (Equiv. Norton):



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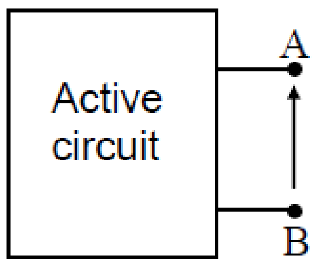
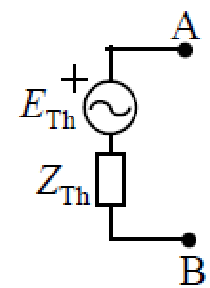
Equivalent Thevenin

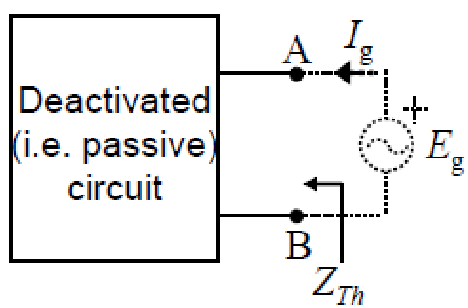
- E_{Th}: is the voltage between A and B in open circuit:



$$V_{AB} = V_A - V_B = E_{Th} \text{ in o.c.}$$



- Z_{Th}: is the equivalent impedance seen trough A-B

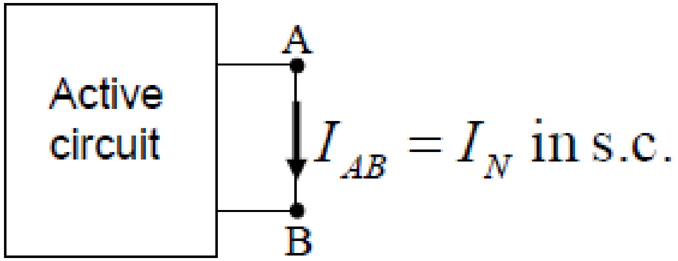


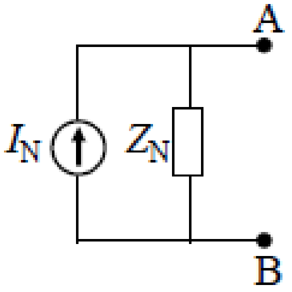
$$Z_{Th} = \frac{E_g}{I_g}, \text{ by annulling}^{(*)} \text{ the independend sourcesin the circuit}$$

(*): Voltage source replaced by sort circuit
Current source replaced by open circuit

Equivalent Norton

- I_N: is the current between A and B in short circuit:





- Z_N (as Z_{Th}) is the equivalent impedance seen trough A-B
- Equivalences between Thevenin and Norton:

$$\begin{cases} E_{Th} = Z_N I_N \\ Z_{Th} = Z_N \end{cases}$$