EXERCISES MODULE 3: SOURCES AND METHODS OF ANALYSIS. THEVENIN's AND NORTON's EQUIVALENT SOURCES.

November 22, 2016

PROBLEM 3.1

Given the circuit of Figure 1:

a) Determine the equations that allow to solve the circuit using the mesh analysis method.

b) Determine the equations that allow to solve the circuit using the nodal analysis method.

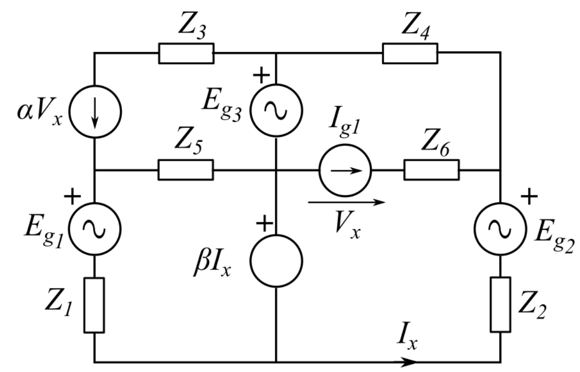

Figure 1

PROBLEM 3.2

Given the circuit in Figure 2:

a) Determine the system of equations that allows to solve the circuit using the mesh analysis method.

b) Determine the system of equations that allows to solve the circuit using the nodal analysis method.

c) Solve the circuit using the equations obtained in a), and determine the potential of points A, B and C (V_A, V_B, V_C).

d) Solve the circuit using the equations obtained in b), and determine the potential of points A, B and C (V_A, V_B, V_C).

e) Do these potentials coincide independently of the method of analysis used to compute them?

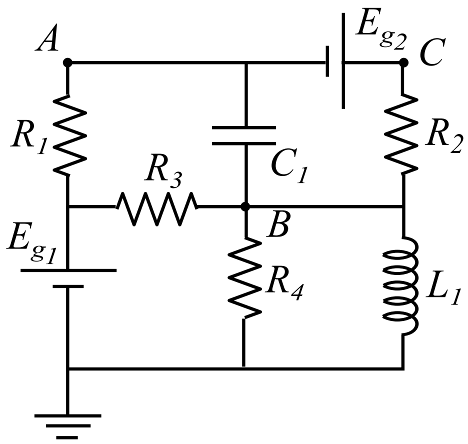

Figure 2

Data:

$$E_{g_1} = 1 \text{ V}; \quad E_{g_2} = 1 \text{ V}$$
$$R_1 = 1\ \Omega; \quad R_2 = 1\ \Omega$$
$$R_3 = 1\ \Omega; \quad R_4 = 2\ \Omega$$
$$C_1 = 1\ \mu\text{F}; \quad L_1 = 1 \text{ mF}$$

Result
$V_A = 0$ V, $V_B = 0$ V, $V_C = 1$ V

PROBLEM 3.3

Given the circuit in Figure 3:

a) Determine the system of equations that allows to solve the circuit using the mesh analysis method.

b) Determine the system of equations that allows to solve the circuit using the nodal analysis method.

c) Solve the circuit using the equations obtained in a), and determine the difference of potentials between points A and B, V_{AB}.

d) Solve the circuit using the equations obtained in b), and determine the difference of potentials between points A and B, V_{AB}.

e) Do these potentials coincide independently of the method of analysis used to compute them?

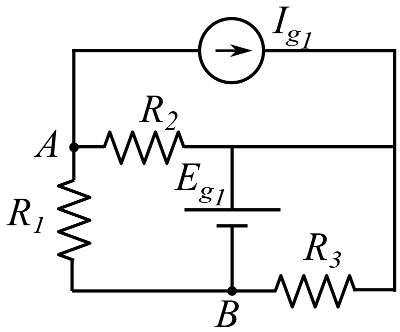

Figure 3

Data:

$E_{g_1} = 3$ V; $I_{g_1} = 6$ A
$R_1 = 1\ \Omega$; $R_2 = 2\ \Omega$
$R_3 = 3\ \Omega$;

Result
$V_{AB} = -3$ V

PROBLEM 3.4

Given the circuit in Figure 4:

a) Determine the system of equations that allows to solve the circuit using the mesh analysis method.

b) Determine the system of equations that allows to solve the circuit using the nodal analysis method.

c) Solve the circuit using the equations obtained in a), and determine the difference of potentials between points B and A, V_{BA}.

d) Solve the circuit using the equations obtained in b), and determine the difference of potentials between points B and A, V_{BA}.

e) Do these difference of potentials coincide independently of the method of analysis used to compute them?

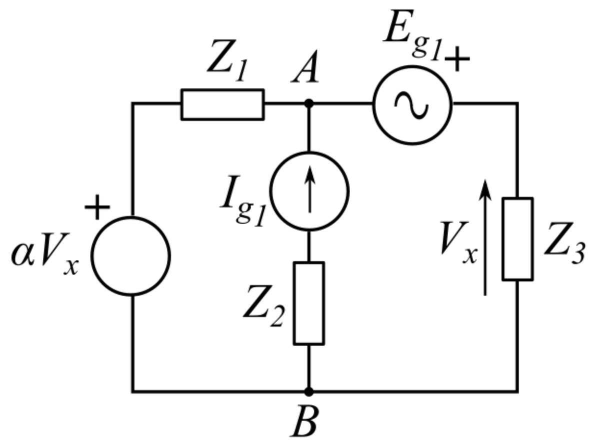

Figure 4

Data:

$$E_{g_1} = 2 + 2j \text{ V}; \quad I_{g_1} = 4j \text{ A}$$
$$Z_1 = j\ \Omega; \quad Z_2 = 1 + j\ \Omega$$
$$Z_3 = 2 - j\ \Omega; \quad \alpha = 2$$

Result
$V_{BA} = 3j$ V

PROBLEM 3.5

Given the circuit in Figure 5:

a) Determine the system of equations that allows to solve the circuit using the mesh analysis method.

b) Determine the system of equations that allows to solve the circuit using the nodal analysis method.

c) Solve the circuit using the equations obtained in a), and determine the difference of potentials between points C and D, V_{CD}.

d) Solve the circuit using the equations obtained in b), and determine the difference of potentials between points C and D, V_{CD}.

e) Do these difference of potentials coincide independently of the method of analysis used to compute them?

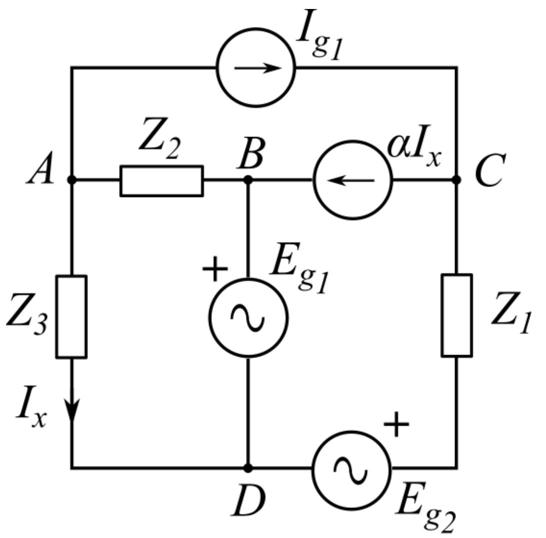

Figure 5

Data:

$$E_{g_1} = 2 + j \text{ V}; \quad E_{g_2} = 1 - 3j \text{ V}$$
$$Z_1 = -j\ \Omega; \quad Z_2 = 1\ \Omega$$
$$Z_3 = 1\ \Omega; \quad I_{g_1} = j \text{ A}$$
$$\alpha = 1;$$

Result

$V_{CD} = 2 - 2j$ V

PROBLEM 3.6

Given the circuit in Figure 6:

a) Determine the system of equations that allows to solve the circuit using the mesh analysis method.

b) Determine the system of equations that allows to solve the circuit using the nodal analysis method.

c) Solve the circuit using the equations obtained in a), and determine the current $i_x(t)$.

d) Solve the circuit using the equations obtained in b), and determine the current $i_x(t)$.

e) Do the expressions for $i_x(t)$ coincide independently of the method of analysis used to obtain them?

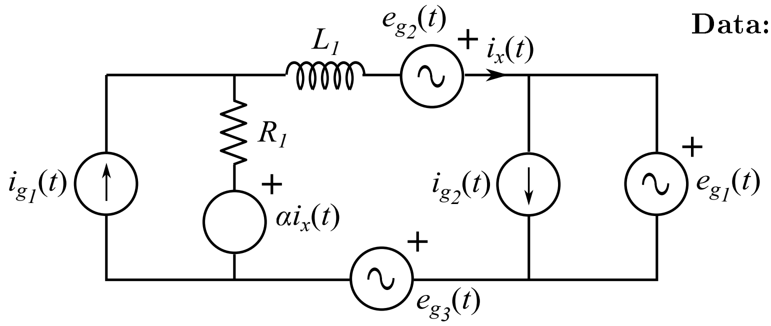

Figure 6

Data:

$$e_{g1}(t) = \cos\left(t - \frac{\pi}{2}\right) \text{ V}$$
$$e_{g2}(t) = 2\cos(t) \text{ V}$$
$$e_{g3}(t) = \cos\left(t + \frac{\pi}{2}\right) \text{ V}$$
$$i_{g1}(t) = 2\cos\left(t + \frac{\pi}{2}\right) \text{ A}$$
$$i_{g2}(t) = \cos\left(t - \frac{\pi}{2}\right) \text{ A}$$
$$R_1 = 1\ \Omega$$
$$L_1 = 1 \text{ H}$$
$$\alpha = 2\ \Omega$$

Result

$i_x(t) = 2\cos\left(t - \frac{\pi}{2}\right)$ A

PROBLEM 3.7

Given the circuit in Figure 7:

a) Determine the system of equations that allows to solve the circuit using the mesh analysis method.

b) Determine the system of equations that allows to solve the circuit using the nodal analysis method.

c) Solve the circuit using the equations obtained in a), and determine the voltage $v_x(t)$.

d) Solve the circuit using the equations obtained in b), and determine the voltage $v_x(t)$.

e) Do the expressions for $v_x(t)$ coincide independently of the method of analysis used to obtain them?

f) Compute the power of the source $i_{g1}(t)$) and the power absorbed by L_1 and R_2.

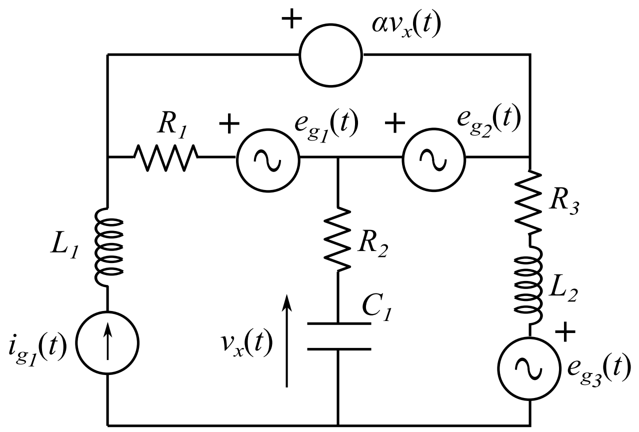

Figure 7

Data:

$$e_{g1}(t) = \sqrt{2}\cos\left(t - \tfrac{\pi}{4}\right)\ \text{V}$$
$$e_{g2}(t) = \cos\left(t + \tfrac{\pi}{2}\right)\ \text{V}$$
$$e_{g3}(t) = \cos\left(t - \tfrac{\pi}{2}\right)\ \text{V}$$
$$i_{g1}(t) = \cos(t)\ \text{A}$$

$R_1 = 1\ \Omega$ $\quad R_2 = 1\ \Omega$

$R_3 = 1\ \Omega$ $\quad C_1 = 1\ \text{F}$

$L_1 = 1\ \text{H}$ $\quad L_2 = 1\ \text{H}$

$\alpha = 1$

Result

$$v_x(t) = \frac{1}{\sqrt{2}}\cos\left(t - \frac{\pi}{4}\right)\ \text{V};\quad P_{I_{g1}} = \frac{3}{4}\ \text{W};\quad P_{Z_{L_1}} = 0\ \text{W};\quad P_{R_2} = \frac{1}{4}\ \text{W}$$

PROBLEM 3.8

Given the circuit in Figure 8:

a) Determine the system of equations that allows to solve the circuit using the mesh analysis method.

b) Determine the system of equations that allows to solve the circuit using the nodal analysis method.

c) Solve the circuit using the equations obtained in a), and determine the difference of potentials between points A and B, V_{AB}.

d) Solve the circuit using the equations obtained in b), and determine the difference of potentials between points A and B, V_{AB}.

e) Do these difference of potentials coincide independently of the method of analysis used to compute them?

f) Compute the power of the source βV_x and the power absorbed by the impedance Z_1.

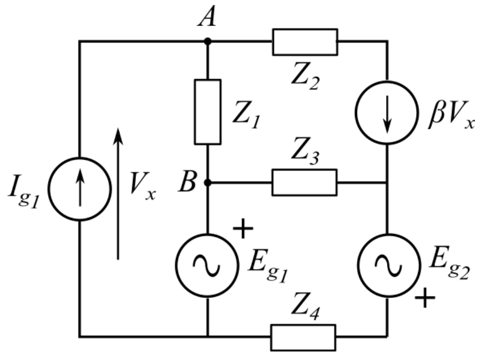

Figure 8

Data:

$$E_{g_1} = 2 + 2j \text{ V}; \quad E_{g_2} = -1 \text{ V}$$
$$I_{g_1} = 2 + j \text{ A};$$
$$Z_1 = 2\ \Omega; \qquad Z_2 = 1 + j\ \Omega$$
$$Z_3 = 2\ \Omega; \qquad Z_4 = -2j\ \Omega$$
$$\beta = \frac{1}{2}\ \Omega^{-1}$$

Result

$V_{AB} = 1$ V; $P_{\beta V_x} = \dfrac{17}{8}$ W; $P_{Z_1} = \dfrac{1}{4}$ W.

PROBLEM 3.9

Determine the Thevenin's equivalent source from terminals A and B for the circuit in Figure 9.

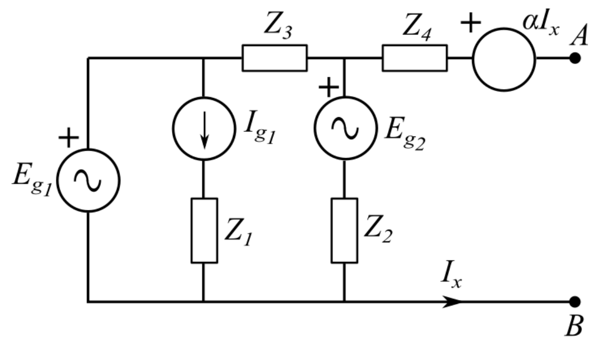

Figure 9

Data:

$$E_{g_1} = 4 + 3j \text{ V}; \quad E_{g_2} = 3 \text{ V}$$
$$I_{g_1} = 1 \text{ A};$$
$$Z_1 = 1\ \Omega; \quad Z_2 = 1 - j\ \Omega$$
$$Z_3 = 1 + j\ \Omega; \quad Z_4 = 1 + j\ \Omega$$
$$\alpha = 1\ \Omega$$

Result
$E_{Th} = 5 + j$ V ; $Z_{Th} = 1 + j\ \Omega$

PROBLEM 3.10

Determine the Thevenin's equivalent source from terminals A and B for the circuit in Figure 10.

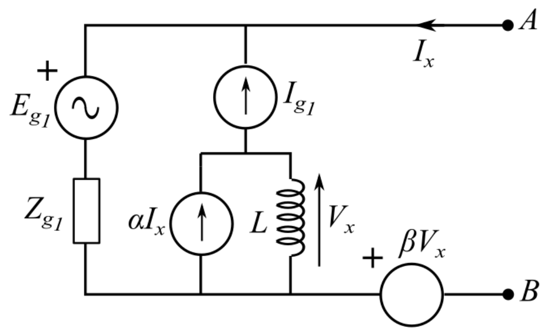

Figure 10

Data:

$$E_{g_1} = 10\sqrt{2} \cdot e^{-j\frac{\pi}{4}} \text{ V}; \quad I_{g_1} = \sqrt{2} \cdot e^{j\frac{\pi}{4}} \text{ A}$$
$$Z_{g_1} = 1 + j\ \Omega; \quad Z_L = j\ \Omega$$
$$\alpha = \frac{1}{2}; \quad \beta = 2$$

Result
$E_{Th} = 12 - 10j$ V; $Z_{Th} = 1 + 2j\ \Omega$

PROBLEM 3.11

Determine the Norton's equivalent source from terminals A and B for the circuit in Figure 11.

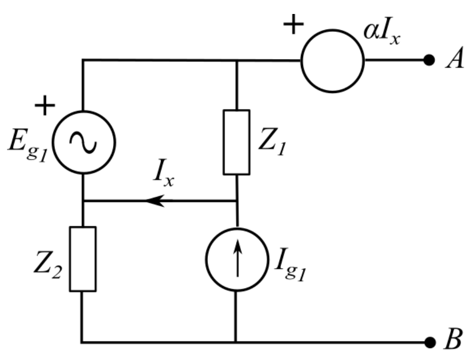

Figure 11

Data:

$$E_{g_1} = 2j \text{ V}; \quad I_{g_1} = 1 \text{ A}$$
$$Z_1 = j\ \Omega; \quad Z_2 = 2 - j\ \Omega$$
$$\alpha = 1\ \Omega$$

Result

$$I_N = \frac{-3+j}{5} \text{ A}; \quad Z_N = 2 - j\ \Omega$$

PROBLEM 3.12

Determine the Thevenin's equivalent source from terminals A and B for the circuit in Figure 12.

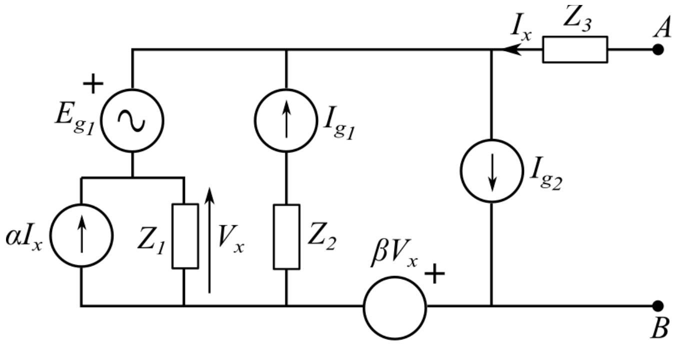

Figure 12

Data:

$E_{g_1} = j$ V; $\quad I_{g_1} = 2$ A;

$I_{g_2} = 1$ A $\quad Z_1 = \frac{1}{2}j\ \Omega$;

$Z_2 = j\ \Omega$; $\quad Z_3 = 2j\ \Omega$

$\alpha = 1$; $\quad \beta = 2$

Result

$$E_{Th} = \frac{j}{2} \text{ V}; \quad Z_{Th} = j\ \Omega$$

PROBLEM 3.13

Determine the Norton's equivalent source from terminals A and B for the circuit in Figure 13.

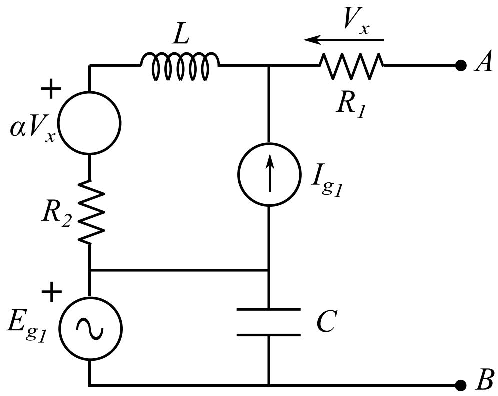

Figure 13

Data:

$E_{g_1} = 2$ V; $\quad I_{g_1} = 2$ A

$R_1 = 1\ \Omega$; $\quad R_2 = 1\ \Omega$

$Z_L = j\ \Omega$; $\quad \alpha = 2$

$Z_C = -j\ \Omega$;

Result

$$I_N = 2 - 4j \text{ A}; \quad Z_N = j\ \Omega$$

PROBLEM 3.14

For the circuit in Figure 14:

a) Determine the Norton's equivalent source from terminals A and B towards the left.

b) Power absorbed by the load R_L.

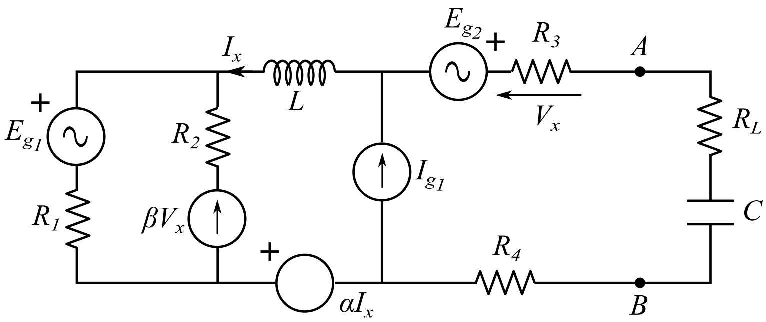

Figure 14

Data:

$$E_{g_1} = 2 \text{ V}; \quad E_{g_2} = 1 - j \text{ V} \quad I_{g_1} = j \text{ A}; \quad R_L = 3\ \Omega; \quad R_1 = R_2 = R_3 = R_4 = 1\ \Omega;$$
$$Z_L = j\ \Omega; \quad Z_C = -j\ \Omega; \quad \alpha = 2\ \Omega; \quad \beta = 2\ \Omega^{-1}$$

Result

a) $I_N = \dfrac{4+2j}{5}$; A $\quad Z_N = (3+j)\ \Omega$.

b) $P_{R_L} = \dfrac{1}{3}$ W

PROBLEM 3.15

For the circuit in Figure 15:

a) Determine the Thevenin's equivalent source from terminals A and B towards the left.

b) Power absorbed by the load R_L.

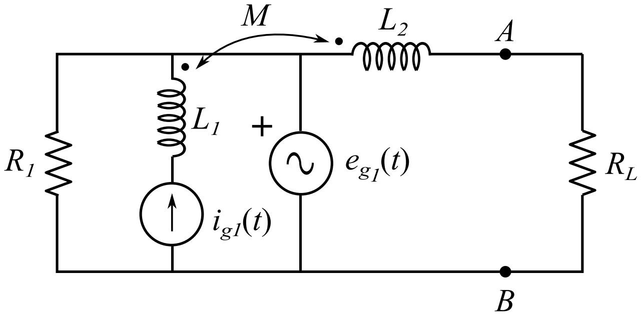

Figure 15

Data:

$$L_1 = 4 \text{ mH}; \quad L_2 = 9 \text{ mH};$$
$$R_L = 45\ \Omega; \quad R_g = 200\ \Omega;$$
$$k = \frac{1}{2}$$

$$i_{g_1}(t) = 2\ cos\left(5 \cdot 10^3 t - \frac{\pi}{2}\right) \text{ A};$$
$$e_{g_1}(t) = 50\ cos\left(5 \cdot 10^3 t\right) \text{ V}.$$

Result

a) $e_{Th}(t) = 80\ cos\left(5 \cdot 10^3 t\right)$ V; $\quad Z_{Th} = 45j\ \Omega$

b) $P_{R_L} = \dfrac{320}{9}$ W

PROBLEM 3.16

In the circuit shown in Figure 16 there is a perfect coupling between the two inductors.

a) Determine the Thevenin's equivalent source from terminals A and B towards the right.

b) Compute the power of the source $e_{g_1}(t)$.

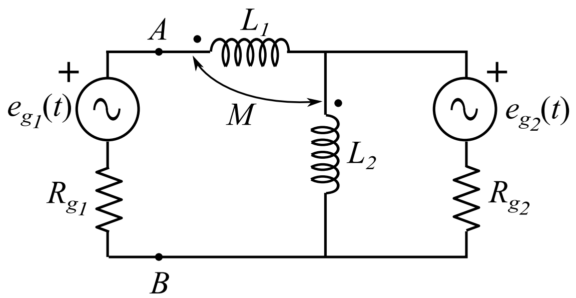

Figure 16

Data:

$$e_{g_1}(t) = 2\sqrt{2}\,\sin\left(10^3 t + \frac{\pi}{4}\right)\ \text{V};$$
$$e_{g_2}(t) = \sin(10^3 t)\ \text{V};$$
$R_{g_1} = 2\ \Omega$; $\quad R_{g_2} = 1\ \Omega$;

$L_1 = L_2 = 1$ mH; $\quad k = 1$

Result

a) $e_{TH}(t) = \sqrt{2}\,\sin\left(10^3 t + \dfrac{\pi}{4}\right)$ V; $\quad Z_{TH} = 2 + 2j\ \Omega$

b) $P_{E_{g_1}} = \dfrac{4}{10}$ W