EXERCISES MODULE 4: THEOREMS OF LINEARITY AND TRANSFORMERS.

December 16, 2016

PROBLEM 4.1

For the circuit in Figure 1:

a) Obtain the value of current $i(t)$.

b) Compute the power absorbed by resistor R.

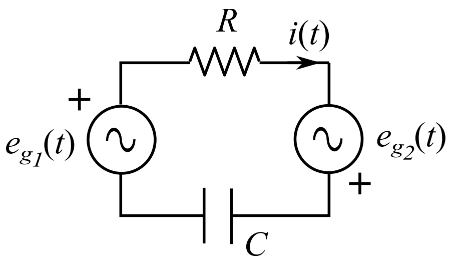

Figure 1

Data:

$$e_{g_1}(t) = \sqrt{2}\sin\left(10^3 t + \frac{\pi}{4}\right)\ \text{V}; \quad R = 1\ \text{k}\Omega$$
$$e_{g_2}(t) = \cos\left(2\cdot 10^3 t\right)\ \text{V}; \quad C = 1\ \mu\text{F}$$

Result

a) $i(t) = \sin\left(10^3 t + \frac{\pi}{2}\right) + \frac{2}{\sqrt{5}}\cos\left(2\cdot 10^3 t + 0{,}464\right)$ mA.

b) $P_R = \frac{9}{10}$ mW

PROBLEM 4.2

For the circuit in Figure 1:

a) Obtain the value of current $i(t)$.

b) Compute the power absorbed by resistor R.

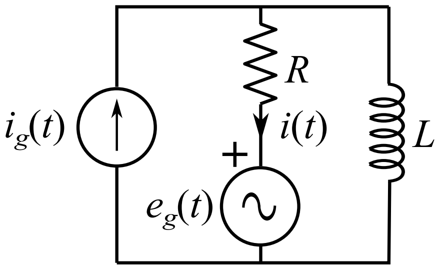

Figure 1

Data:

$$e_g(t) = 5\sqrt{2}\,\sin\left(2\cdot 10^6 t + \frac{\pi}{4}\right)\ \text{V};\quad R = 1\ \text{k}\Omega$$
$$i_g(t) = 5\,\cos\left(\frac{1}{2}\cdot 10^6\ t\right)\ \text{mA};\quad L = 1\ \text{mH}$$

Result

a) $i(t) = \sqrt{10}\,\sin\left(2\cdot 10^6 t + 2{,}82\right) + \sqrt{5}\ \cos\left(\frac{1}{2}\cdot 10^6 t + 1{,}107\right)\ \text{mA}.$

b) $P_R = \dfrac{15}{2}\ \text{mW}$

PROBLEM 4.3

For the circuit in Figure 1:

a) Obtain the current $i_x(t)$.

b) Compute the power absorbed by resistor R.

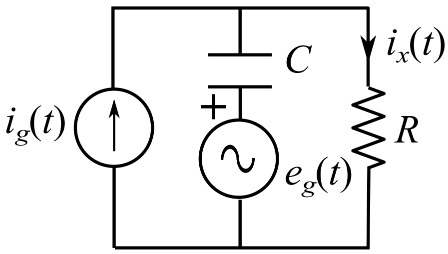

Figure 1

Data:

$$i_g(t) = 5\sqrt{2}\,\sin\left(2\cdot 10^3 t + \frac{\pi}{4}\right)\ \text{mA};\quad R = 2\ \text{k}\Omega$$
$$e_g(t) = \sqrt{2}\,\sin\left(10^3\ t - \frac{\pi}{4}\right)\ \text{V};\quad C = \frac{1}{2}\ \mu\text{F}$$

Result

a) $i_x(t) = \sqrt{10}\,\sin\left(2\cdot 10^3 t - 0.346\right) + \frac{1}{2}\,\sin\left(10^3\ t\right)\ \text{mA}.$

b) $P_R = \dfrac{41}{4}\ \text{mW}$

PROBLEM 4.4

For the circuit in Figure 1:

a) Obtain the expression for current $i_x(t)$.

b) Compute the power of the sources $\alpha v_x(t)$ and E_{g3}, and the power absorbed by resistor R_2.

c) Determine the new value of current $i_x(t)$, if we substitute the source $e_{g_1}(t)$ for a different source $e'_{g_1}(t) = 2\sqrt{2}\cos\left(10^6\, t + \frac{\pi}{4}\right)$ V

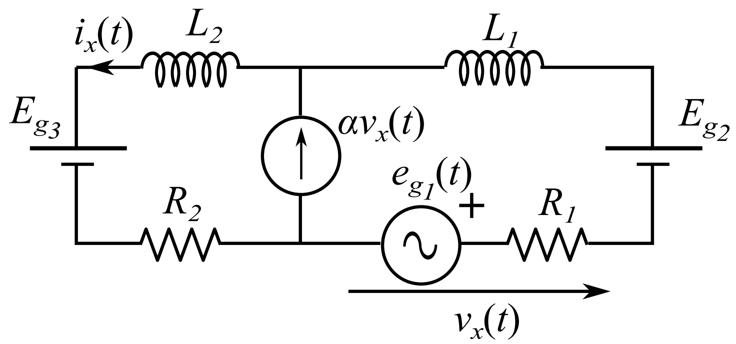

Figure 1

Data:

$$R_1 = 1\ \text{k}\Omega; \qquad R_2 = 2\ \text{k}\Omega;$$
$$L_1 = 1\ \text{mH}; \qquad L_2 = 3\ \text{mH}$$
$$E_{g2} = 3\ \text{V}; \qquad E_{g3} = 2\ \text{V}$$
$$\alpha = 2\ \text{m}\Omega^{-1}; \quad e_{g_1}(t) = 4\sqrt{2}\cos\left(10^6\, t + \frac{\pi}{4}\right)\ \text{V}.$$

Result

a) $i_x(t) = 1 + 4\sqrt{2}\cos\left(10^6 t - \frac{3\pi}{4}\right)$ mA.

b) $P_{\alpha V_x} = 136$ mW; $\quad P_{E_{g3}} = -2$ mW; $\quad P_{R_2} = 34$ mW

c) $i'_x(t) = 1 + 2\sqrt{2}\cos\left(10^6 t - \frac{3\pi}{4}\right)$ mA.

PROBLEM 4.5

For the circuit in Figure 1:

a) Determine the expression of current $i_1(t)$.

b) Compute the power for the sources $\alpha i_x(t)$ and E_g, and the power absorbed by resistor R_2.

c) If we substitute the source E_g for a novel source $E'_g = 6$ V, obtain the new value of the power absorbed by resistor R_2.

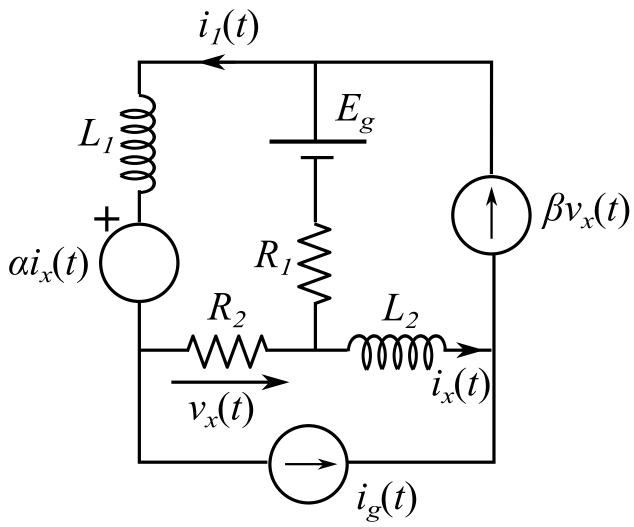

Figure 1

Data:

$R_1 = 1\ \text{k}\Omega;$	$R_2 = 2\ \text{k}\Omega;$
$L_1 = 3\ \text{mH};$	$L_2 = 2\ \text{mH}$
$E_g = 3\ \text{V};$	$\beta = 2\ \text{m}\Omega^{-1}$
$\alpha = 1\ \text{k}\Omega;$	$i_g(t) = 2\ \sin\left(10^6\ t\right)\ \text{mA}.$

Result

a) $i_1(t) = 1 + \sqrt{2}\,\sin\left(10^6 t - \frac{\pi}{4}\right)\ \text{mA}.$

b) $P_{\alpha I_x} = 5\ \text{mW};\quad P_{E_g} = 15\ \text{mW};\quad P_{R_2} = 4\ \text{mW}$

c) $P'_{R_2} = 10\ \text{mW}$

PROBLEM 4.6

For the circuit in Figure 1:

a) Obtain the expression of current $i_1(t)$.

b) Determine the power of sources $\alpha v_x(t)$, E_{g2} and $e_{g_1}(t)$, and the power absorbed by resistor R. Verify the power balance principle.

c) If we substitute source $e_{g_1}(t)$ for a new source $e'_{g_1}(t) = 4\ \cos\left(10^6\ t\right)\ \text{V}$, obtain the new power for the source $\alpha v_x(t)$.

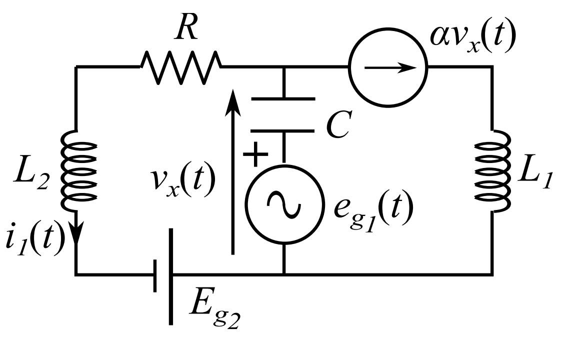

Figure 1

Data:

$R = 2\ \text{k}\Omega;$	$\alpha = 2\ \text{m}\Omega^{-1};$	
$L_1 = 2\ \text{mH};$	$L_2 = 1\ \text{mH};$	$C = 1; \text{nF}$
$E_{g_2} = 5\ \text{V};$	$e_{g_1}(t) = 8\ \cos\left(10^6\ t\right)\ \text{V}.$	

Result

a) $i_1(t) = 2 + \sqrt{2}\cos\left(10^6 t + \frac{\pi}{4}\right)$ mA.

b) $P_{\alpha V_x} = -12$ mW; $P_{E_{g_2}} = 10$ mW; $P_{e_{g_1}(t)} = 12$ mW; $P_{R_1} = 10$ mW

c) $P'_{\alpha V_x} = -4.5$ mW

PROBLEM 4.7

For the circuit in Figure 1:

a) Determine the current $i_1(t)$.

b) Compute the power of the source $\alpha v_x(t)$, and the power absorbed by resistor R_2.

c) If we substitute sources $i_g(t)$, $e_{g_1}(t)$ and $e_{g_2}(t)$ for the following new sources:

$$\begin{cases} i'_g(t) = 2\cos\left(10^6\ t + \frac{\pi}{2}\right)\ \text{mA;} \\ e'_{g_1}(t) = 4\cos\left(10^6\ t\right)\ \text{V;} \\ e'_{g_2}(t) = 2\cos\left(10^6\ t\right)\ \text{V} \end{cases}$$

Obtain the new value of the power of the dependent source $\alpha v_x(t)$.

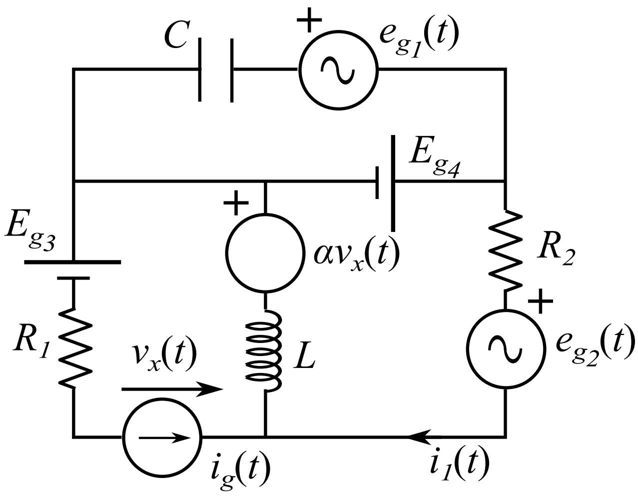

Figure 1

Data:

$R_1 = 1$ kΩ; $R_2 = \frac{1}{2}$ kΩ

$\alpha = 1$; $L = 1$ mH;

$C = 2$ nF; $E_{g_3} = 2$ V

$E_{g_4} = 1$ V; $e_{g_1}(t) = 2\cos\left(10^6\ t\right)$ V;

$e_{g_2}(t) = \cos\left(10^6\ t\right)$ V; $i_g(t) = \cos\left(10^6\ t + \frac{\pi}{2}\right)$ mA.

Result

a) $i_1(t) = 4 + \cos\left(10^6 t + \frac{\pi}{2}\right)$ mA.

b) $P_{\alpha V_x} = 4.5$ mW; $P_{R_2} = 8.25$ mW

c) $P'_{\alpha V_x} = 6$ mW

PROBLEM 4.8

For the circuit in Figure 1 obtain the power of sources $e_g(t)$ and $i(t)$.

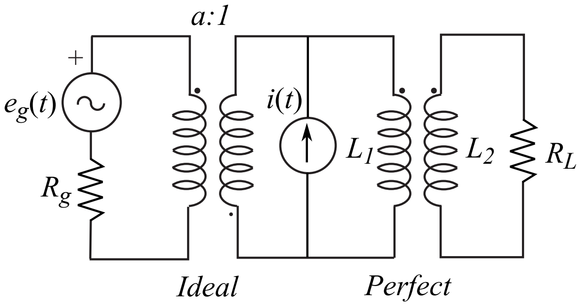

Figure 1

$$Data: \quad e_g(t) = 4\sin(\omega t)\ V \quad ; \quad i(t) = 2\sin(\omega t)\ A \quad ; \quad R_g = 2\ \Omega \quad ; \quad R_L = 2\ \Omega$$
$$\omega L_1 = \frac{1}{4}\ \Omega \quad ; \quad \omega L_2 = 1\ \Omega \quad ; \quad a = 2$$

Result

$P_{E_g} = \frac{7}{2}\ W$ (delivered); $P_I = -\frac{1}{4}\ W$ (absorbed).

PROBLEM 4.9

For the circuit in Figure 1, obtain the Thevenin's equivalent source from terminals A and B.

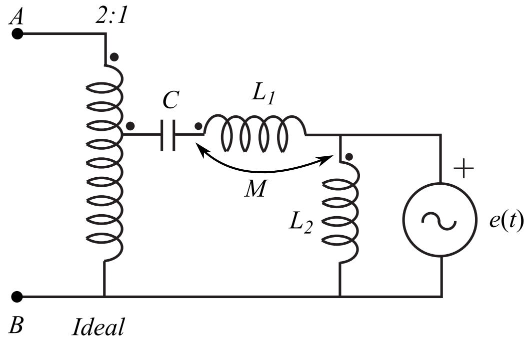

Figure 1

$$Data: \quad e\,(t) = 3\ sen\,(t)\ V \quad ; \quad C = 4\ F \quad ; \quad L_1 = 4\ H \quad ; \quad L_2 = 1\ H \quad ; \quad K = 0.5$$

Result

$e_{Th}(t) = 12\sin(t)$ V; $Z_{Th} = 11j\ \Omega$.

PROBLEM 4.10

For the circuit in Figure 1:

a) Obtain the Thevenin's equivalent source from terminals $A - B$ **towards the left**.

b) Detemirne the equivalent impedance (Z_s) from terminals $A - B$ **towards the right** .

c) Obtain the power absorbed by resistor R.

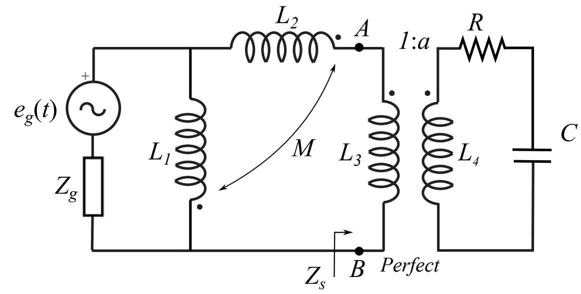

Figure 1

Data:

$$e_g(t) = 5\ \sin\left(10^3 t\right)\ \text{V};\quad L_1 = L_2 = 2\ \text{mH};\quad L_3 = 1\ \text{mH};\quad a = \sqrt{10}$$
$$Z_g = (1 - 2j)\ \Omega;\quad k_{12} = \frac{1}{2};\quad R = 1\ \Omega;\quad C = \frac{1}{7}\ \text{mF}$$

Result

a) $e_{Th}(t) = 5\ \sin\left(10^3 t + \frac{\pi}{2}\right)$ V; $Z_{Th} = (1 + 2j)\ \Omega$.

b) $Z_s = (1 - 2j)\ \Omega$.

c) $P_R = \frac{25}{8}$ W

PROBLEM 4.11

For the circuit in Figure 1 we know that the coupling coefficient for L_p and L_s is $k_{ps} = \frac{1}{\sqrt{2}}$.

a) Compute the equivalent impedance of the circuit from terminals $C - D$ towards the right.

b) Obtain the Thevenin's equivalent source from terminals $A - B$ towards the right.

c) Determine the power absorbed by sources $e(t)$ and $i(t)$.

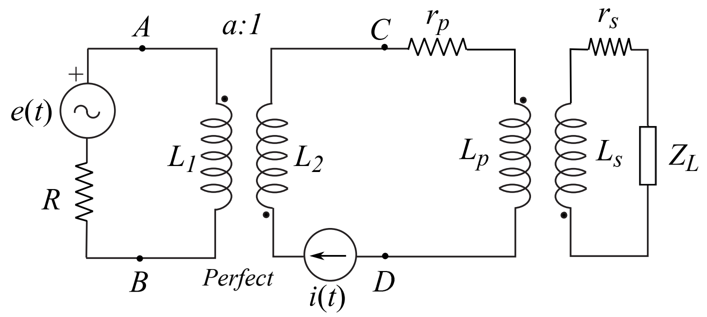

Figure 1

Data:

$$e(t) = 4\ \cos(\omega t)\ \text{V};\quad \omega L_1 = 2\ \Omega;\quad \omega L_2 = 8\ \Omega;\quad \omega L_p = \omega L_s = 2\ \Omega;\quad Z_L = -2j\ \Omega$$
$$i(t) = 4\ \cos\left(\omega t - \frac{\pi}{2}\right)\ \text{A};\quad r_p = 1\ \Omega;\quad r_s = 2\ \Omega;\quad R = 2\ \Omega$$

Result

a) $Z_{CD} = (2 + 2j)\ \Omega$.

b) $e_{Th}(t) = 16 \cdot \cos(\omega t)$ V, $Z_{Th} = 2j\ \Omega$.

c) $P_E = -6$ W, $P_I = 40$ W.

PROBLEM 4.12

Obtain the Thevenin's equivalent source $(e_{Th}(t), Z_{Th})$ from terminals $A - B$ for the circuit shown in Figure 1.

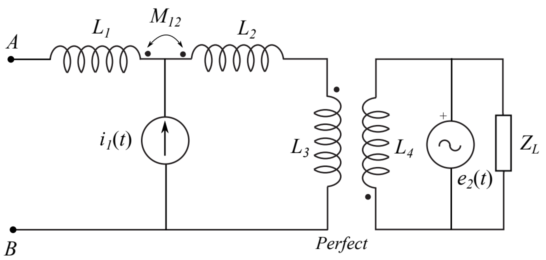

Figure 1

Data:

$i(t) = \sin\left(10^6 t\right)$ A; $e(t) = 2\sin\left(10^6 t\right)$ V; $L_1 = 1\ \mu$H; $L_2 = 4\ \mu$H

$L_3 = 4\ \mu$H; $L_4 = 1\ \mu$H; $k_{12} = \frac{1}{2}$; $Z_L = 100\ \Omega$

Result

$e_{Th}(t) = 5\sin\left(10^6 t + 2.4981\right)$ V; $Z_{Th} = 3j\ \Omega$.

PROBLEM 4.13

For the circuit shown in Figure 1:

a) Determine the equivalent impedance from terminals AB towards the right, Z_{AB}.

b) Obtain the time-domain expression for the current passing through resistor R_2, $i(t)$.

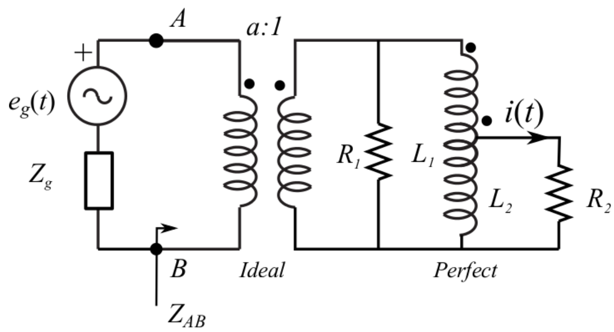

Figure 1

Data:

$$e_g(t) = 12\cos\left(10^3 t\right)\ \text{V};\quad Z_g = 9+9j\ \Omega;\quad L_1 = \frac{9}{2}\ \text{mH};\quad L_2 = \frac{1}{2}\ \text{mH}$$
$$a = 2 \qquad R_1 = 9\ \Omega; \qquad R_2 = 1\ \Omega$$

Result

a) $Z_{AB} = 9(1+j)\ \Omega$.

b) $i(t) = \cos\left(10^3 t\right)\ \text{A}$

PROBLEM 4.14

For the circuit shown in Figure 1, obtain the power of the source $e_g(t)$.

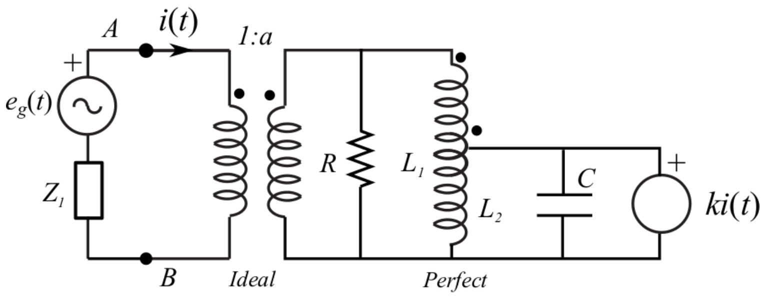

Figure 1

Data:

$$e_g(t) = 12\cos\left(10^6 t\right)\ \text{V};\quad Z_1 = 6(1-2j)\ \Omega;\quad C = 7\ \mu\text{F};\quad L_1 = 3\ \mu\text{H};$$
$$L_2 = \frac{1}{3}\ \mu\text{H}; \qquad a = 2; \qquad k = 4\ \Omega; \qquad R = 9\ \Omega.$$

Result

$P_{E_g} = 3\ \text{W}$