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Main Menu

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A new experimental method for calculating K_I and K_{II} using photoelasticity

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ABSTRACT

A novel experimental approach for the calculation of stress intensity factors from the analysis of photoelastic images is presented. The method is derived from the well-established six image phase-stepping method but only two images are used to obtain the retardation angle. To calculate K_I and K_{II} a set of discrete data points are collected from the map of retardation angle in the region surrounding the crack tip. A theoretical description of the spatial variation of retardation angle is derived from Westergaard's model and fitted to the data via an error function using a least squares fitting procedure. In this way, the necessity of unwrapping is avoided and the consequential difficulties and errors are eliminated. To demonstrate the efficacy of the proposed approach, stress intensity values have been obtained from photoelastic images captured during a test conducted using a polycarbonate Center-Cracked Tension (CCT) specimen. The results are very promising, showing a high level of agreement with those predicted from theory.

Keywords: Fatigue crack, stress intensity factor, photoelasticity, image analysis

INTRODUCTION

Accurate crack detection and fatigue damage assessment in industrial components have focused the attention of many researchers for a long time. In this context, the ability to make reliable stress measurements at a crack tip is an essential part in understanding the fatigue process.

Since Post and Wells [1, 2] first showed the potential of photoelasticity for fracture mechanics applications in the 1950's, many authors have focused their research on applying fracture mechanics to structural analysis. One major step forward in the calculation of the stress intensity factor from the analysis of photoelastic images was made by Sanford and Dally [3] with their Multi-Point Over-Deterministic Method (MPODM). Since then, this approach has been extensively used to report stress intensity factor results for a multiplicity of fracture mechanics problems. Moreover, the method has been successfully extended to other experimental techniques [4-5] as a standard and straightforward methodology for calculating fracture parameters.

Although the MPODM is robust and simple-to-use, it suffers from a substantial limitation in that the absolute value of the isochromatic fringe order must be known at all of the data points utilised in the analysis. This implies either the use of points from the centres of integer or half-order fringes obtained from dark-field or light-field circular polariscope respectively with manual identification of the fringe order [6]; or the use of an unwrapping algorithm [7]. The latter approach has become more popular recently; however both approaches can be difficult to implement successfully in regions of high magnitude and gradient of stress and hence fringe density around a crack. This is particularly problematic for cracks in complex industrial components when the operator must have some skill and previous knowledge to successfully infer the stress intensity factor.

Improvements in computer technology have made it possible to develop image processing methods that have lead to, what nowadays is known as, digital photoelasticity. Different image processing techniques have made it possible to obtain full-field maps of stresses in components by processing photoelastic images [7]. In particular, phase-stepping methods are amongst the most extensively used in fringe image processing to extract phase information. By employing unwrapping techniques, continuous fringe order maps can be obtained from periodic distributions of fringe order, or retardation, evaluated from the phase data. In this way, unwrapping techniques have increased the potential and utility of the initial method proposed by Sanford and Dally [3]. Thus data for use in the MPODM can be sampled from anywhere in the image or even the whole image could be utilised. Clearly the process of unwrapping prior to applying the MPODM adds both time and complexity to the evaluation of a stress intensity factor; hence it would advantageous if wrapped fringe or relative retardation data could be used to solve for stress intensity factors.

In this paper, a new method for calculating stress intensity factors from the analysis of isochromatic data is proposed. The method is based on the determination of the distribution of the relative retardation angle, by combining two photoelastic images, and comparing it with corresponding distribution obtained from a theoretical model. In this way, the unwrapping process is avoided as discussed above, and data points can be collected from the whole image. The results demonstrate that it is a very robust method that provides accurate stress intensity factor values in just a few seconds. To show the potential of the proposed methodology, the method has been used to calculate the stress intensity factor at increasing load steps using a polycarbonate Centred-Cracked-Tension (CCT) specimen with a 10 mm crack. The results show a high level of agreement with those predicted from theory, demonstrating the viability and potential of the proposed methodology.

METHODOLOGY AND EXPERIMENTAL APPROACH

The proposed approach is based on the calculation of the retardation from the analysis of two photoelastic images. Two images are collected from a circular polariscope set up for dark-field and light-field. In these arrangements the light intensity recorded by a digital camera viewing the polariscope can be described by:

$$\begin{aligned} I_1 &= i_m + i_v \cos(\alpha_{\exp}) \\ I_2 &= i_m - i_v \cos(\alpha_{\exp}) \end{aligned} \tag{1}$$

where i_v is the intensity emerging when all the axis of the polariscope and specimen are parallel, i_m takes account stray light [8]. According to equation (1), the retardation angle from experiment, α_{exp} can be obtained experimentally by subtracting image 1 from image 2; therefore:

$$\alpha_{\exp} = \cos^{-1}\left(\frac{I_1 - I_2}{2 i_v}\right) \quad \text{with} \quad \alpha = 2\pi N \tag{2}$$

where N denotes the fringe order.

Hence it is also possible to calculate the retardation angle, α using any one of a number of models for the stress field around a crack tip that are available in the literature [9]. In this case, the Westergaard model [10] has been employed to illustrate the method as shown in equation 3.

$$\begin{aligned} \left(\frac{N f}{t}\right)^2 &= (2\pi r)^{-1}\left[(K_I \cos\theta + 2K_{II}\cos\theta)^2 + (K_{II}\sin\theta)^2\right] + \\ &+ 2\sigma_{0x}(2\pi r)^{-1/2}\sin\frac{\theta}{2}\left[K_I \sin\theta(1 + 2\cos\theta) + K_{II}\left(1 + 2\cos^2\theta + \cos\theta\right)\right] + \sigma_{0x}^2 \end{aligned} \tag{3}$$

where f is the material fringe constant, t is the length of the light path in the photoelastic material r and θ are polar coordinates in a conventional polar system centered at the crack tip with x being in the direction of the crack growth.

In the proposed method, a mathematical fit of the theoretical distribution of retardation angle, α_{theo} is performed to the experimental data, α_{exp} and hence K_I and K_{II} are inferred. The procedure has been divided into six steps (figure 1) that are described in the following paragraphs.

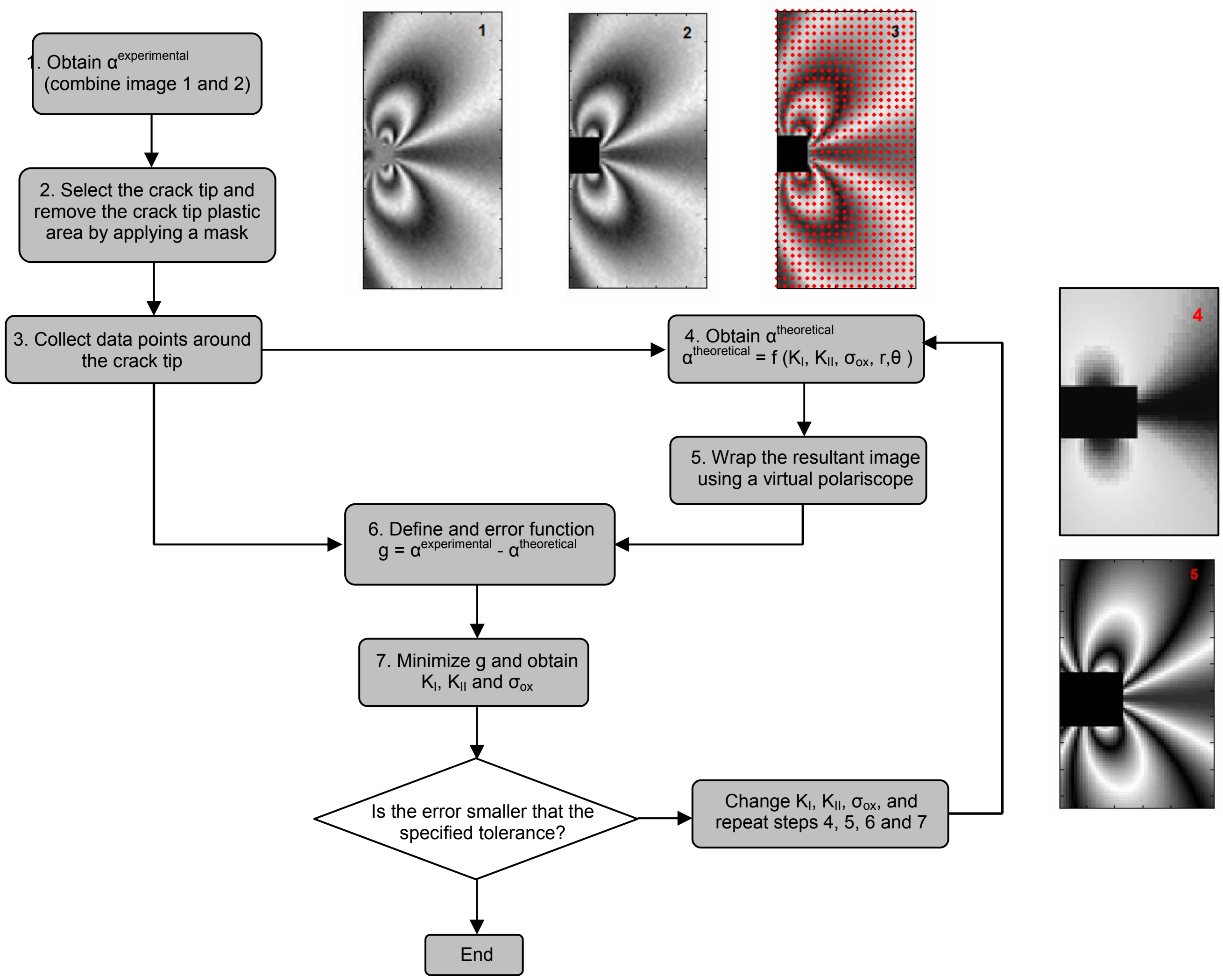

Figure I. Flow chart illustrating the steps of the proposed algorithm for calculating the stress intensity factor from the analysis of isochromatic data.

Initially, the experimental retardation angle is obtained by combining images 1 and 2 as previously indicated (step 1). Subsequently, the crack tip position must be identified by visually inspecting the resultant image (step 2).

To ensure the validity of the adopted mathematical model, based on linear elastic fracture mechanics, a mask must be applied in the near-tip region and along the flanks. The purpose is to remove from the analysis all of the plastic zone around the crack tip. After that, a set of data points are sampled in the region surrounding the crack tip (step 3). The size of the region in which valid data exists was defined by Nurse and Patterson [6] as an annulus of inner radius equal to ten times the crack tip radius and outer radius of approximately 0.4 times the crack length. However, in the proposed method a rectangular area within these limits has been adopted.

The coordinates of the sampled data are then employed to evaluate equation (3) and obtain a theoretical image for the retardation angle according to Westergaard model (step 4). To do this, initial values for K_I and K_{II} have to be defined in this step. The resultant image is continuous since it is directly obtained from the theoretical fringe order. Nevertheless, the experimental image is wrapped and consequently the two distributions cannot be compared directly. To overcome this problem the theoretical distribution is wrapped by using a Matlab routine (step 5) based on Jones matrices to simulate a virtual polariscope [11]. In this way, a map of α_{theo} is generated and can be compared directly to the experimental data for α_{exp}. To calculate K_I, K_{II} and σ_{ox} an error function is defined:

$$g = \alpha_{\exp} - \alpha_{theo} \tag{4}$$

or

$$g(K_I, K_{II}, \sigma_{ox}) = \cos^{-1}\left(\frac{(I_1)_{\exp} - (I_2)_{\exp}}{2 i_v}\right) - \\ - \cos^{-1}\left(\frac{I_1(K_I, K_{II}, \sigma_{ox}, r, \theta) - I_2(K_I, K_{II}, \sigma_{ox}, r, \theta)}{2 i_v}\right) \tag{5}$$

A least squares fit is performed (step 6); since, the resultant system of equations is not linear it has to be solve in an iterative way. For this purpose the Downhill Simplex method was adopted [12].

Finally, K_I and K_{II} values are obtained. The quality of the fitting is assessed by two statistical values, namely the mean and the variance of the least-squared difference between the theoretical and experimental values of retardation angle, α at the sampled data points.

EXPERIMENTAL SETUP

To assess the quality of the proposed methodology experiments were conducted using a 2 mm thick polycarbonate Centre-Cracked-Tension (CCT) specimen with an initial 10 mm centre notch. A crack was created using a razor blade.

To load the specimen, a 30 kN electro-mechanical testing machine (INSTRON 5567) controlled by a desktop computer (Dell - Pentium 4 Intel processor) was employed. During the test, photoelastic images were captured at 100 N increments of load from 0 to 700 N (figure 2).

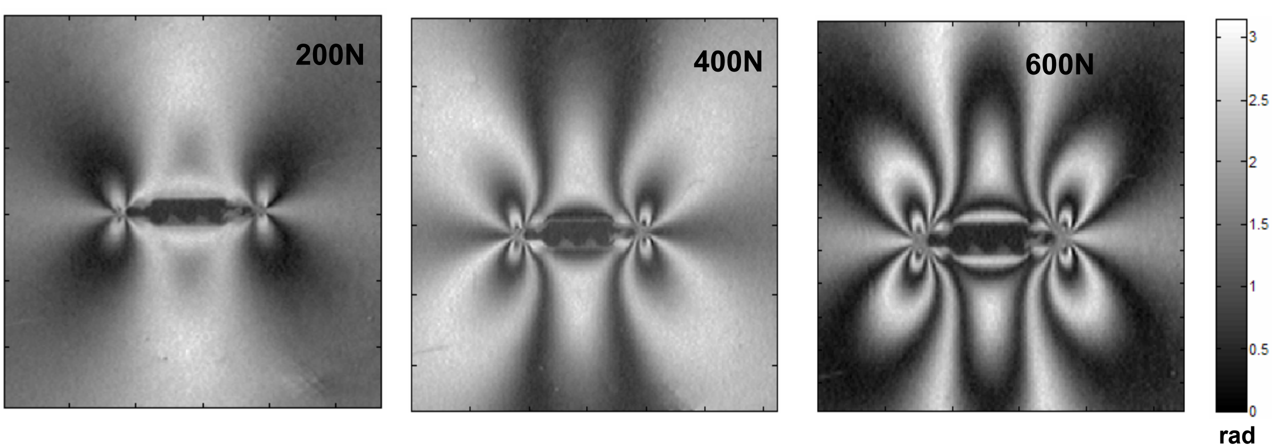

Figure 2. Grey-scale maps of the retardation calculated using the proposed methodology (eq. 2) from photoelastic images captured at 200 N, 400 N and 600 N for 2a = 10 mm.

For this purpose, a transmission polariscope with a monochromatic light source was employed. To capture the images a CCD camera Panasonic VW-BP100 controlled by a laptop (DELL - Centrino Intel Mobile Technology processor) using PC-MCIA video card (Imperx Inc. VCE-B5A01) was used. In addition, a 70-210 mm zoom lens (Tamron model 58A) was employed to increase the spatial resolution in the region of the crack tip.

Subsequently, images were analyzed using a Matlab® computer program implementing the steps described in the previous section and figure 2. As a result, values of K_I, K_{II} and were obtained (figure 3) and compared with those predicted from theory [9]:

$$K_I = C\sigma\sqrt{\pi a} \text{ with } C = 1 + 0.256\left(\frac{a}{W}\right) - 1.152\left(\frac{a}{W}\right)^2 + 12.200\left(\frac{a}{W}\right)^3 \tag{6}$$

RESULTS AND DISCUSSIONS

Figure 3 shows experimental values of K_I and K_{II} inferred from photoelastic images using the procedure described previously and compared with those predicted by theory using equation (6).

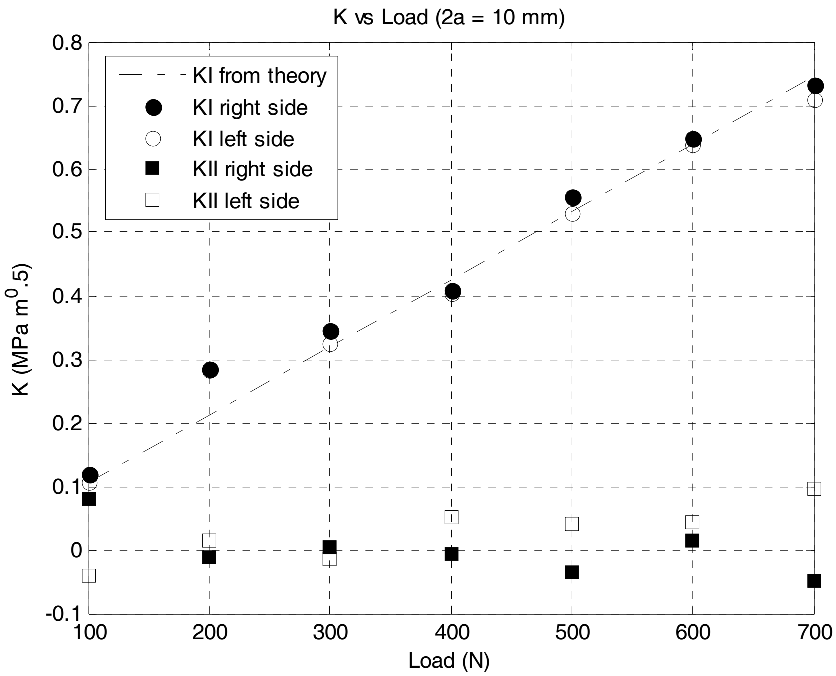

Figure 3. K_I and K_{II} inferred using the proposed methodology from photoelastic images as a function of load for 2a = 10 mm.

Experimental results show an excellent level of agreement with those predicted from theory. There is some scatter in the results which could be attributed to the presence of noise in the images. Another possible reason could be errors introduced when locating the crack tip position, since this was achieved manually by direct observation of the fringe image. Nevertheless in all the cases, the quality of the fitting is characterised by very small values for the normalized mean error (less than 6.25×10^{-3}) and the variance (less than 2.4×10^{-3}). This is also supported by the data in figure 4 which shows a comparison between the experimental retardation and the retardation obtained from the Westergaard model using the values of K_I and K_{II} obtaining by performing the mathematical fitting. As can be observed the images are practically identical with just localised small differences. These differences are reasonable since the adopted analytical solution is based on linear elastic fracture mechanics and assumes a crack in an infinite plate.

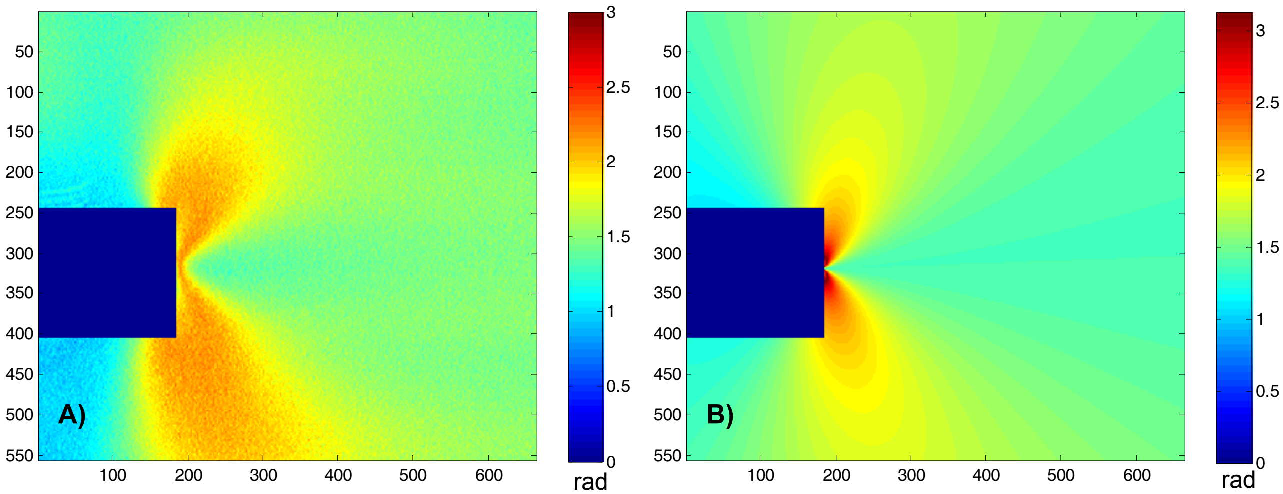

Figure 4. Maps of the photoelastic retardation from a 10 mm crack a) from experiment by combining light- and dark-field circular polariscope images, and b) calculated using Westergaard's model after performing the mathematical fitting.

CONCLUSIONS

A new method for calculating stress intensity factors by the analysis of photoelastic images has been presented. The method makes it possible to obtain accurate results in just a few seconds by capturing only two photoelastic images at different orientations of the analyzer. The new methodology avoids the necessity of unwrapping the phase to calculate the stress intensity factors hence eliminating a potential source of large errors, and is both computational and experimentally efficient. The approach has been tested using a 2 mm thick polycarbonate CCT specimen. Results show excellent agreement with those predicted from theory highlighting the potential of the methodology for other engineering problems.

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