# Determination of isoclinic map for complex photoelastic fringe patterns

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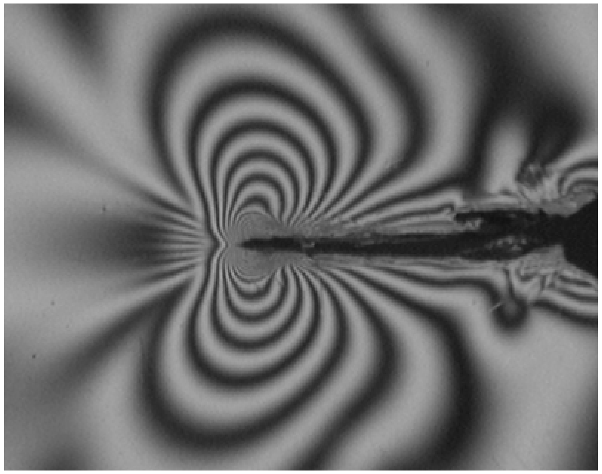
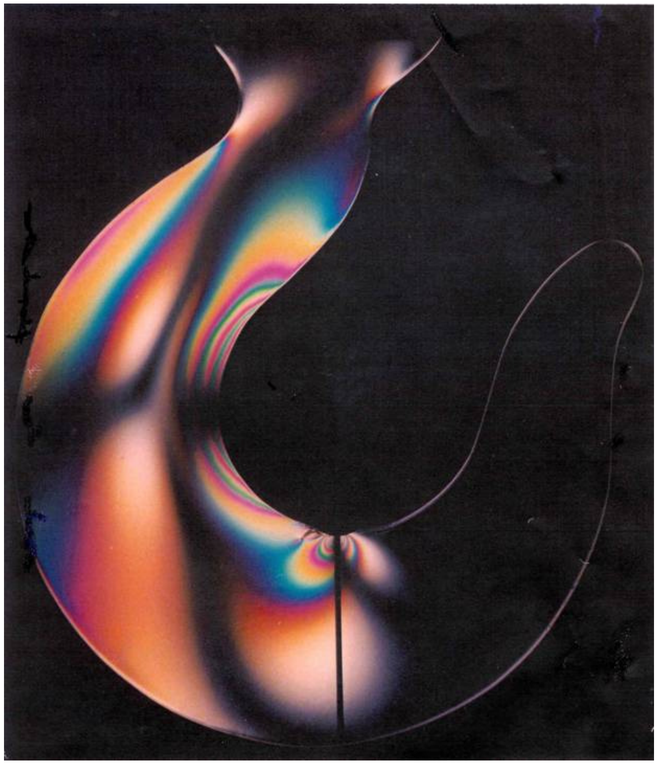
# Outline

- Introduction to photoelastic stress analysis
- Basics of phase-stepping photoelasticity
- Ambiguity of isochromatic fringe maps
- Regularization algorithm and cost function
- Fatigue crack propagation exemplar
- Conclusions



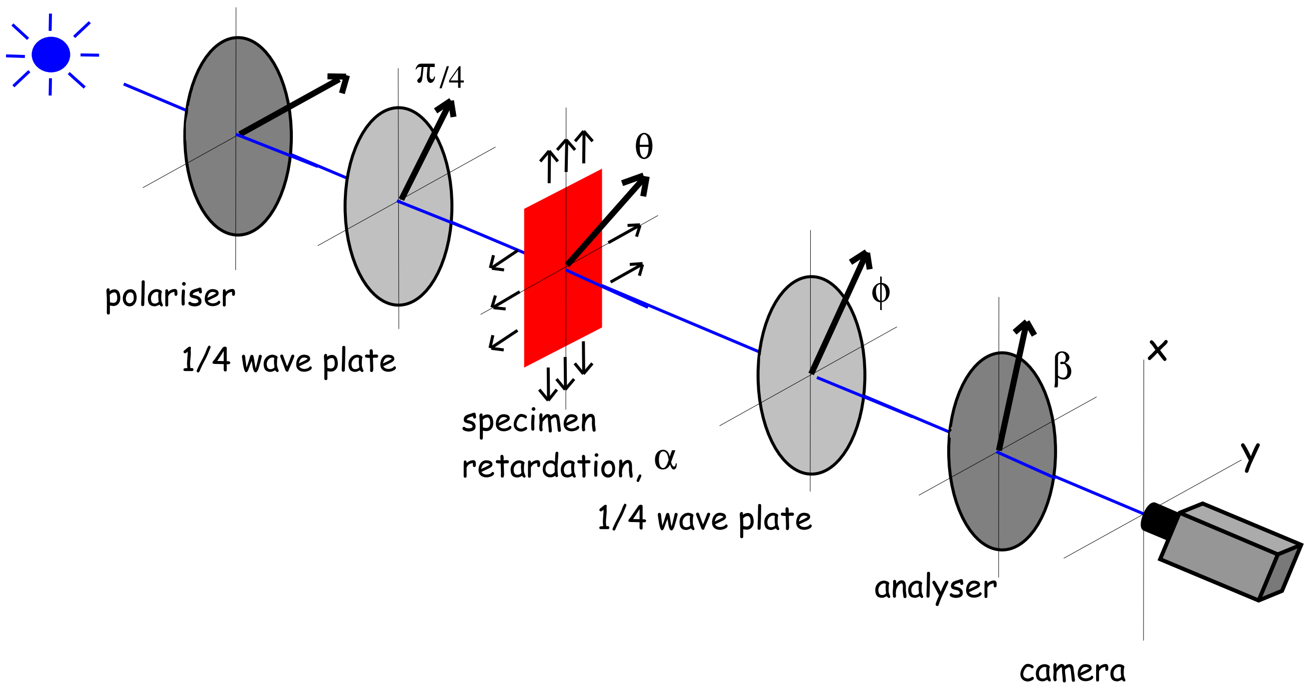
# Introduction

- **All transparent materials subject to stress exhibit temporary birefringence**
  - Isochromatic fringes related to stress magnitude
  - Isoclinic fringes related to direction of principal stresses



- **Variety of digital fringe processing**
  - RGB photoelasticity
    - ☑ No user-input for calibration of fringe map
    - ☒ Limited to isochromatic fringe orders less than 3
  - Fourier transforms
    - ☑ High-quality isochromatic and isoclinic data
    - ☒ Requires tens of images
    - ☒ Fringe unwrapping and user-calibration
  - Phase-stepping
    - ☑ 4 to 6 images for isochromatic & isoclinic data
    - ☒ Fringe unwrapping and user-calibration

# Mathematical description of polariscope




In a circular polariscope, light intensity at a point (x,y):

$$i(x,y) = i_m + i_v\,[\sin 2(\beta-\phi).\cos\alpha - \sin(\theta-\phi).\cos(\beta-\phi).\sin\alpha]$$

Wang, Z.F., Patteson, E.A., Use of phase-stepping with demodulation and fuzzy sets for birefringence measurement, *Optics & Laser Engineering*, 21:91-104, 1995.

# Approach to phase-stepping solution

In a circular polariscope, light intensity at a point $(x,y)$:

$$i(x,y) = i_m + i_v\,[\sin 2(\beta - \phi).\cos\alpha - \sin(\theta - \phi).\cos(\beta - \phi).\sin\alpha]$$

where

$$\alpha = 2\pi N = 2\pi\,(\sigma_1 - \sigma_2)t/f$$

**Unknowns:**

**$\alpha$, relative retardation;**

**$\theta$, isoclinic angle**

**$i_m$ stray light term;**

**$i_v$ constant of proportionality.**

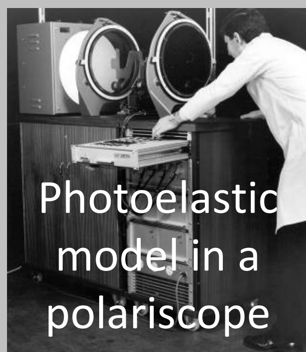
**Variables**

**$\beta$, output $\lambda/4$ plate angle;**

**$\phi$, analyser angle,**

Wang, Z.F., Patteson, E.A., Use of phase-stepping with demodulation and fuzzy sets for birefringence measurement, *Optics & Laser Engineering*, 21:91-104, 1995.

# Fringe processing




## Digital images collected with phase-steps

$$i_1 = i_m + i_v \cos\alpha$$

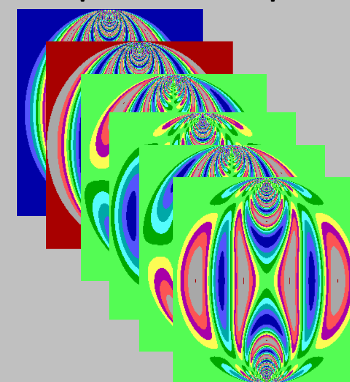
$$i_2 = i_m - i_v \cos\alpha$$

$$i_3 = i_m - i_v \sin\alpha \sin 2\theta$$

$$i_4 = i_m + i_v \sin\alpha \cos 2\theta$$

$$i_5 = i_m + i_v \sin\alpha \sin 2\theta$$

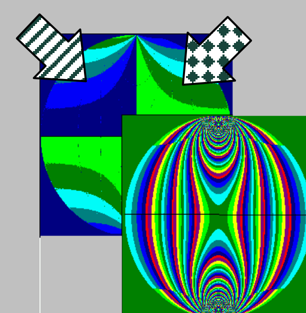
$$i_6 = i_m - i_v \sin\alpha \cos 2\theta$$



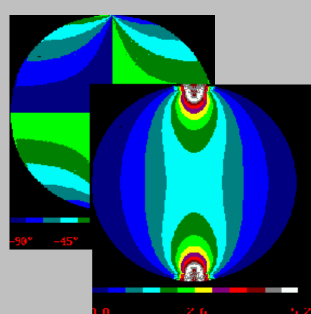
## Periodic isoclinics & isochromatics calculated

$$\theta = \frac{1}{2}\arctan\left[\frac{i_5 - i_3}{i_4 - i_6}\right]$$

$$\alpha = \arctan\left[\frac{i_4 - i_6}{(i_1 - i_2)\cos 2\theta}\right]$$



## Isoclinic & fringe order maps

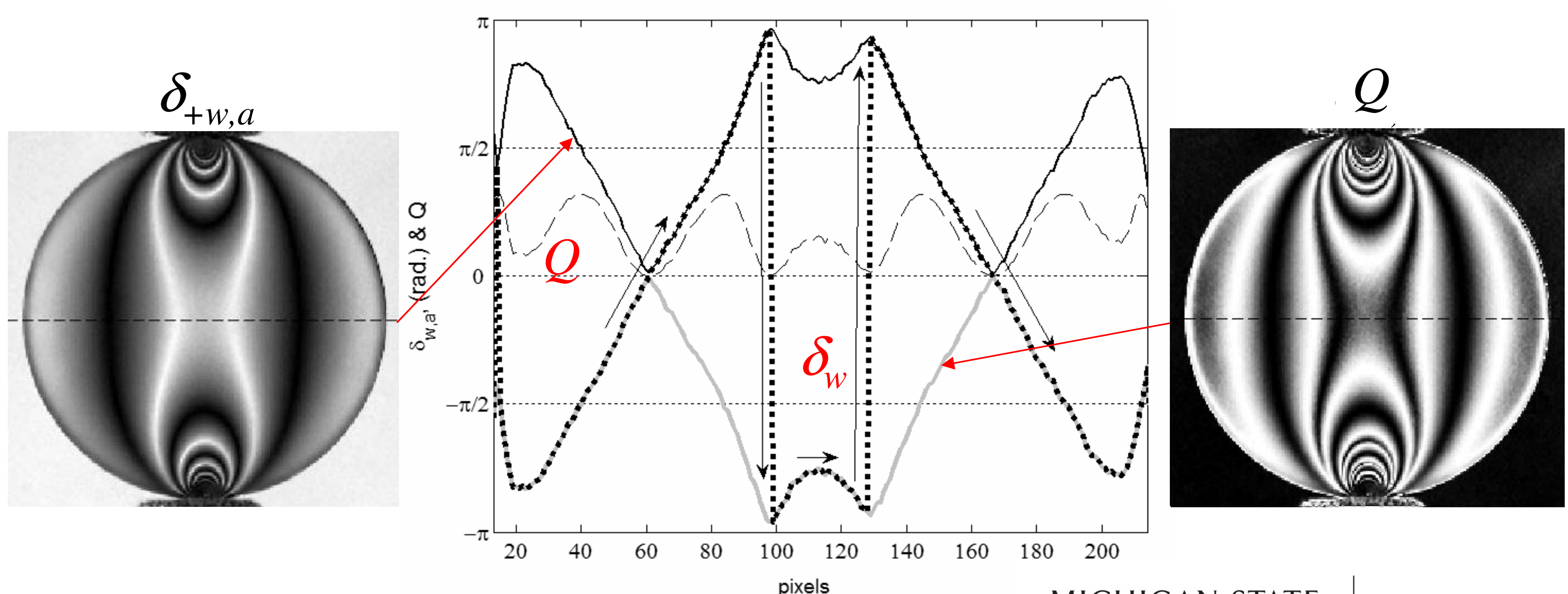


Wang, Z.F., Patteson, E.A., Use of phase-stepping with demodulation and fuzzy sets for birefringence measurement, *Optics & Laser Engineering*, 21:91-104, 1995.

# Ambiguity

- **Small modification to algorithm**
  - Use 4-quadrant arctangent function in MatLab
  - Accounts for sign to yield arctangent between π and - π

$$\theta_a = \frac{1}{2}\arctan_f\left(\frac{I_5 - I_3}{I_4 - I_6}\right) \qquad \delta_{w,a} = \arctan_f\left[\frac{(I_5 - I_3)\sin 2\theta_a + (I_4 - I_6)\cos 2\theta_a}{I_1 - I_2}\right]$$



# Regularization

- **Selection of $\delta_{+w,a}$ or $\delta_{-w,a}$ at each pixel using a regularization algorithm**
  - Start at an arbitrary pixel
  - At following pixel selection depends on $\delta_w$ calculated for neighboring pixels
  - If $-\pi < \delta_{\pm w,a} < 0$ or $\pi > \delta_{\pm w,a} > 0$ then $\delta_w$ must be continuous relative to its neighbors
  - If $\delta_{\pm w,a} = 0$ or $\delta_{\pm w,a} = \pm\pi$ then gradient of $\delta_w$ must be continuous relative to its neighbors

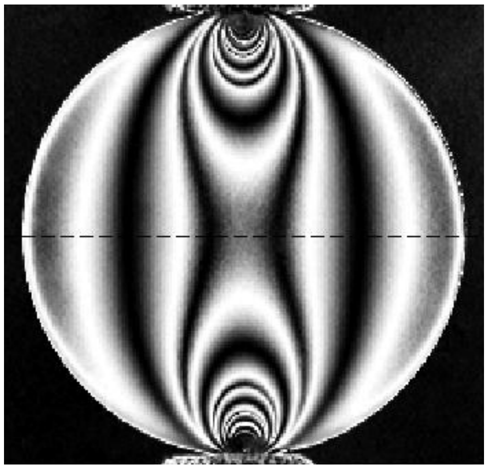
- **Decision implemented using a cost function**

$$U_\Gamma(\mathbf{i};\lambda) = Q(\mathbf{i})A_\Gamma(\mathbf{i}) + \frac{10^\lambda}{Q(\mathbf{i})}B_\Gamma(\mathbf{i})$$

where $\Gamma$ is the window of pixels at $i(x,y)$ being considered

$\lambda$ is a regularization parameter

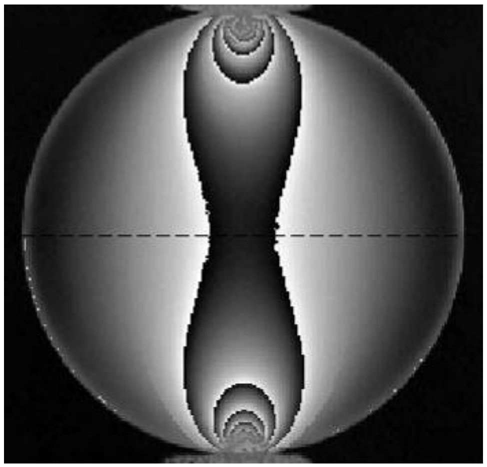
$Q$

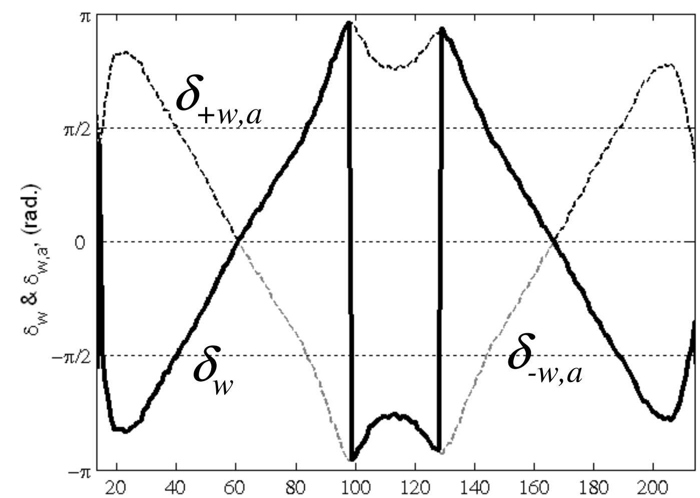


Siegmann, P., Diaz-Garrido, F.A., Patterson, E.A., Robust approach to regularize an isochromatic fringe map, *Applied Optics*, 48(22):E24-E34, 2009

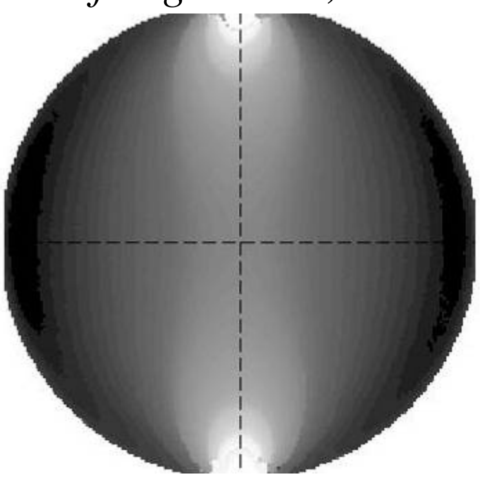
# Application of regularisation

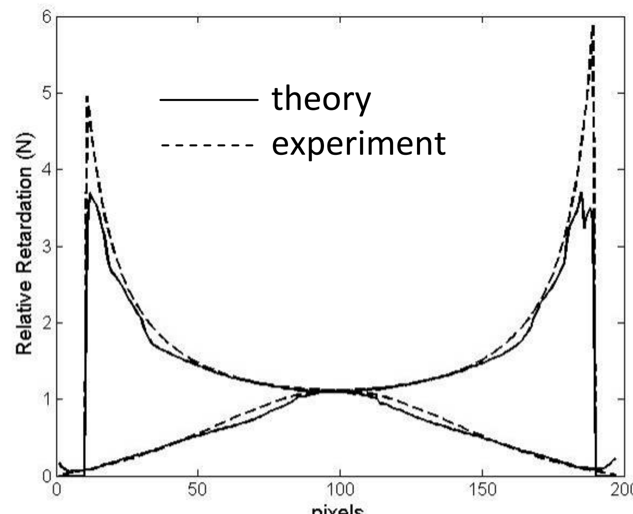
$\delta_w$






*fringe order, N*



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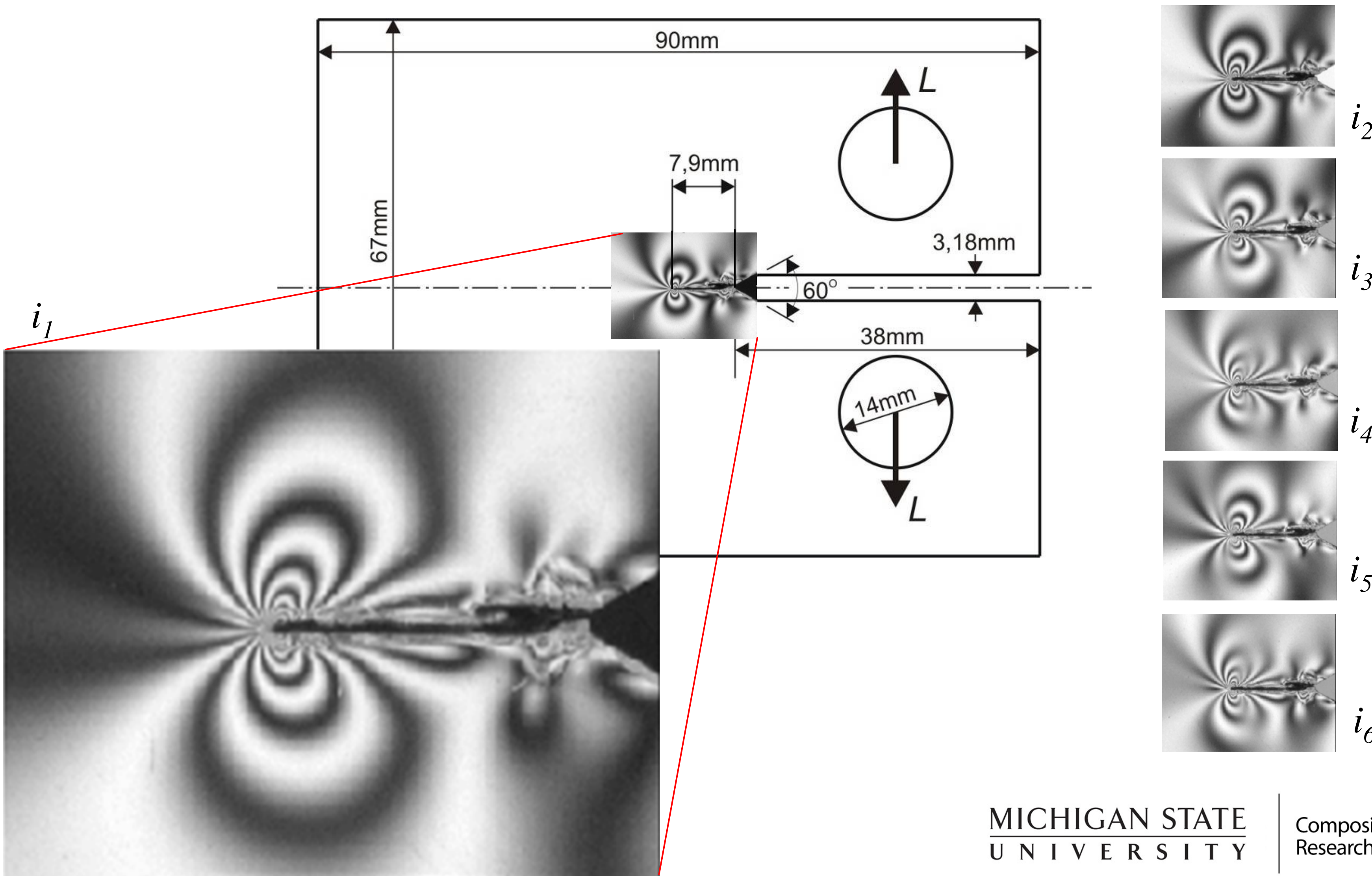
# Crack tip fringe patterns

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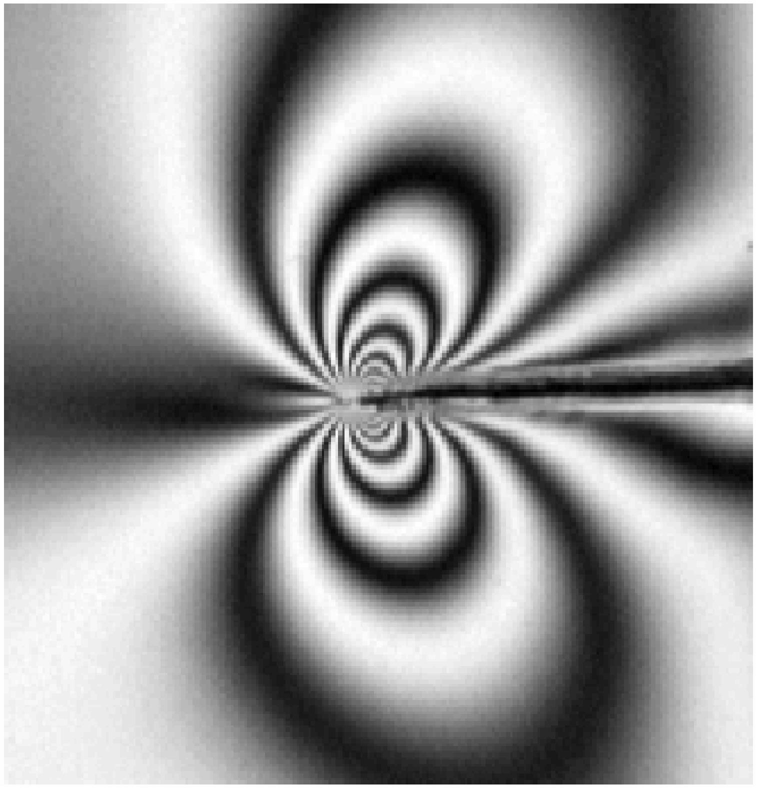
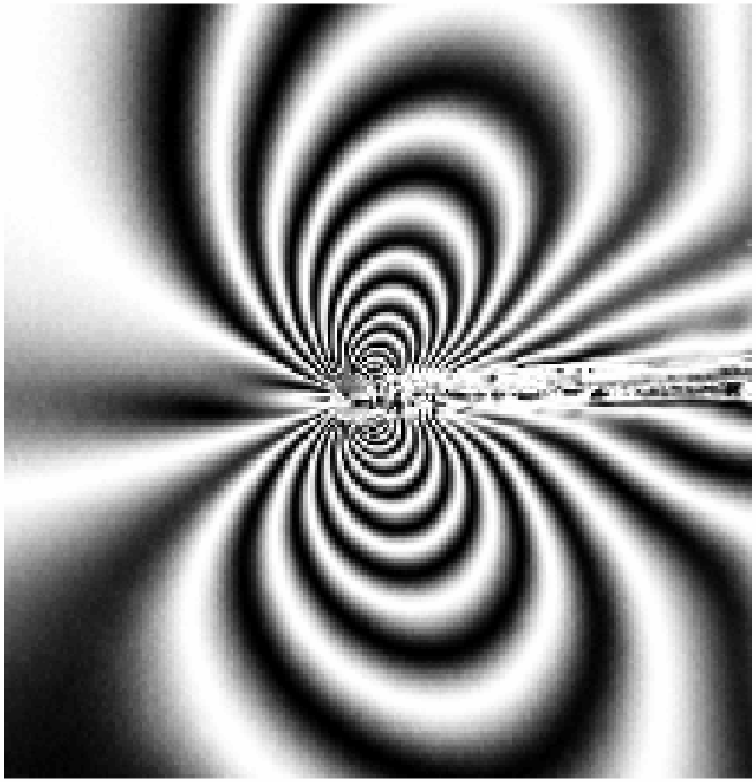
# Crack tip data: pre-regularization

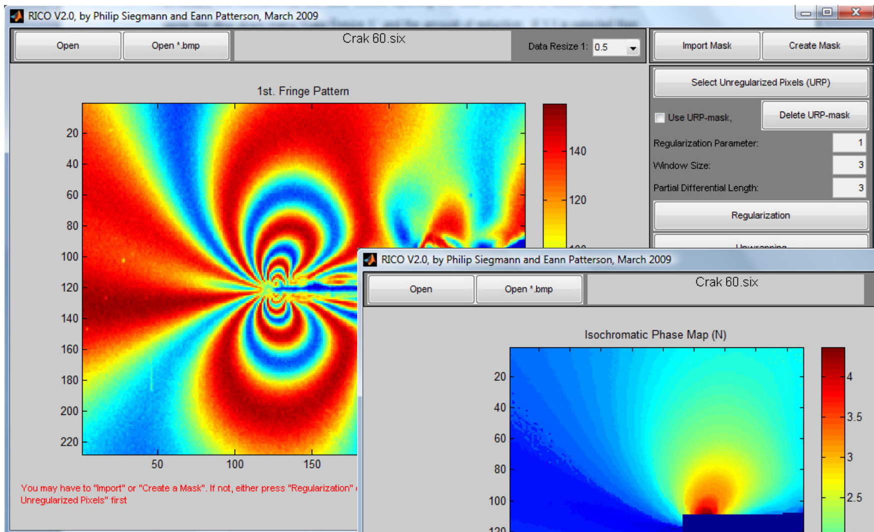
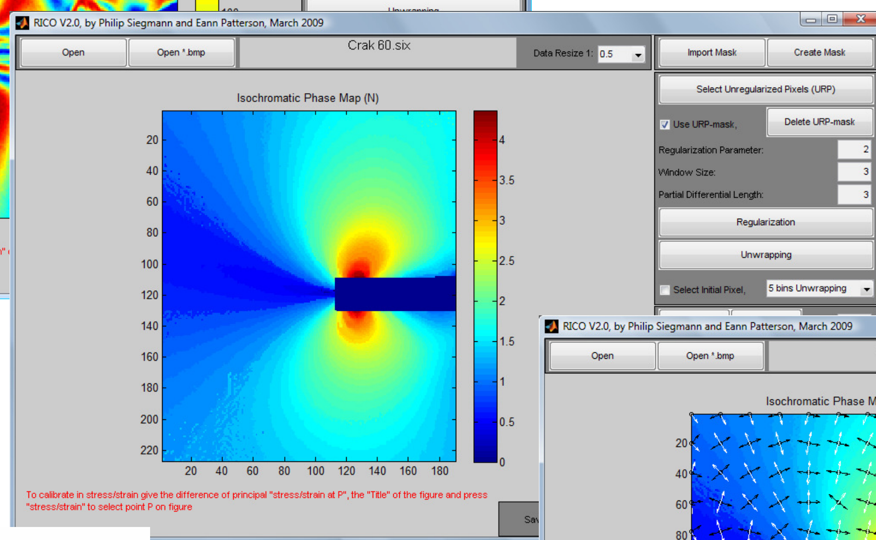
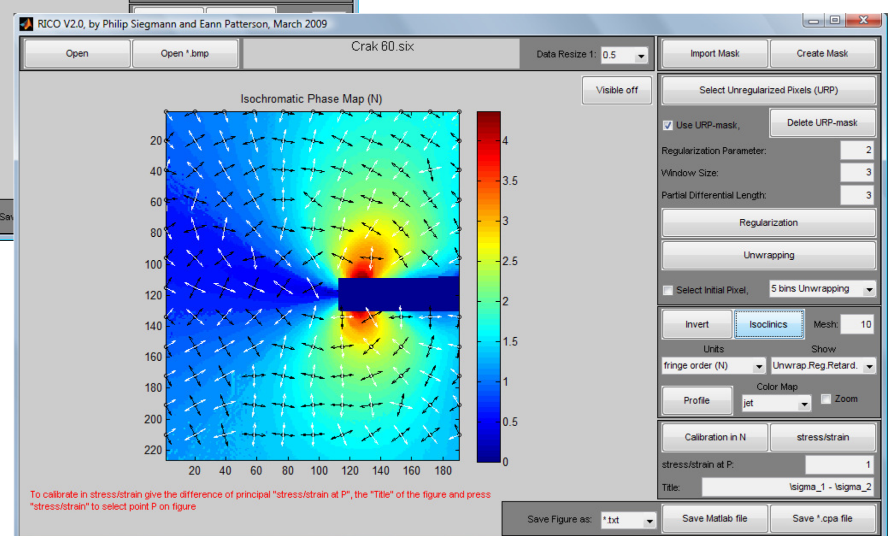
$\delta_{+w,a}$

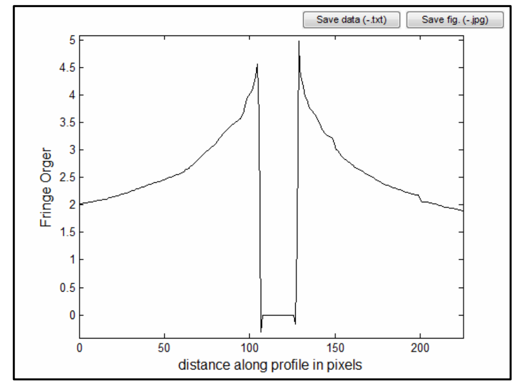


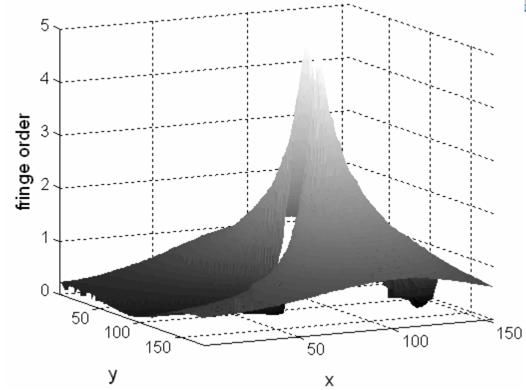
$Q$



# Graphic User Interface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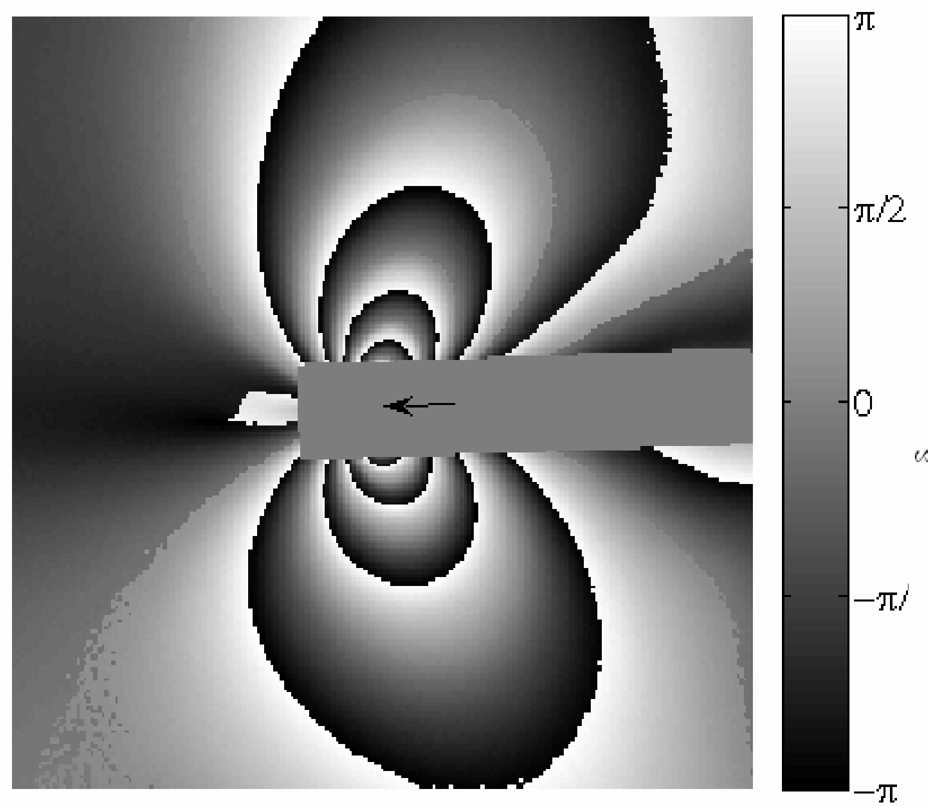
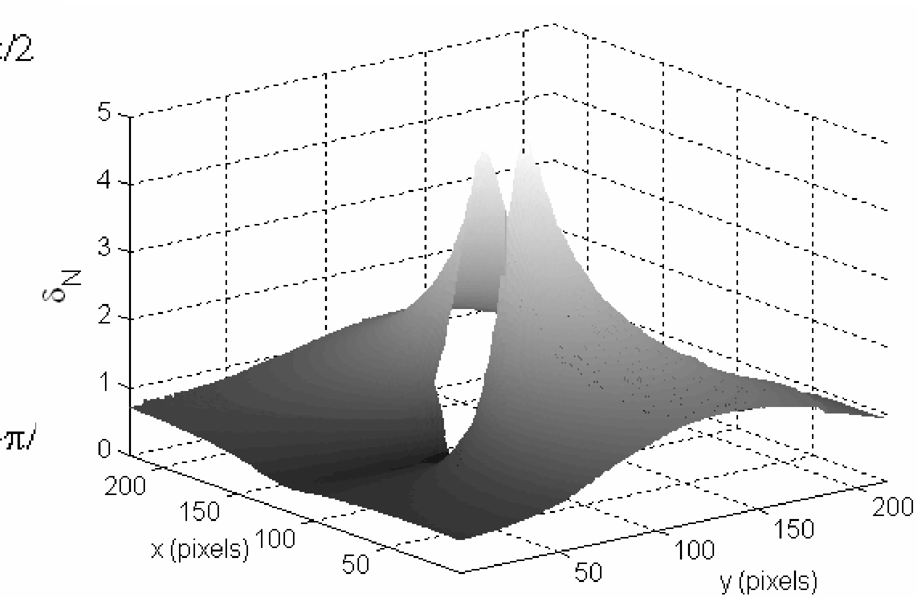
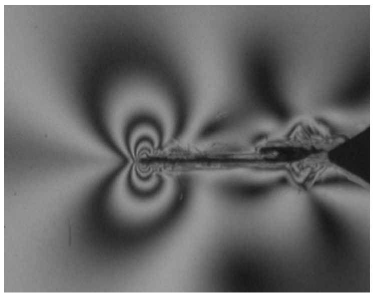
# Crack tip fringe orders

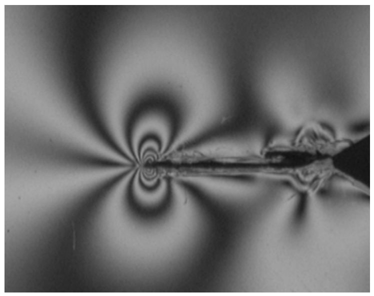


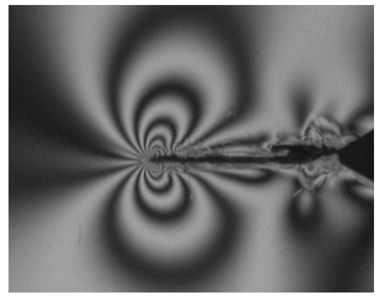
# Plastic shielding investigation



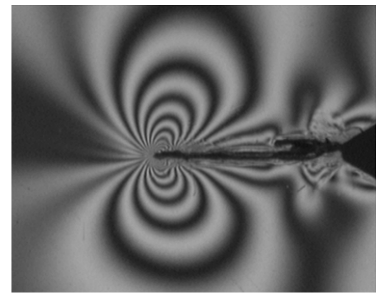
10N



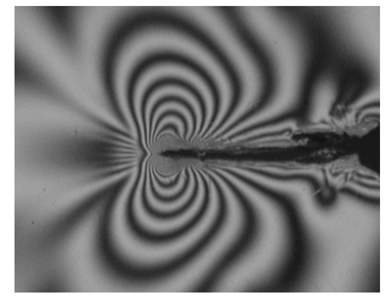
55N



100N

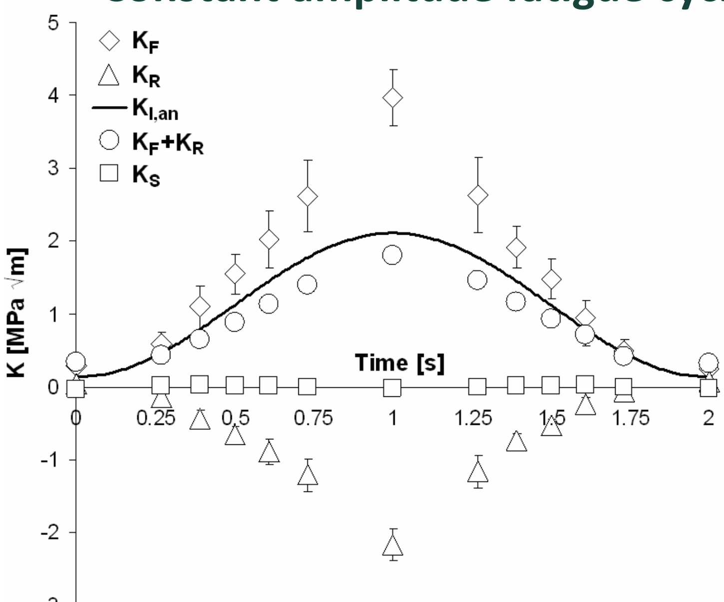


145N

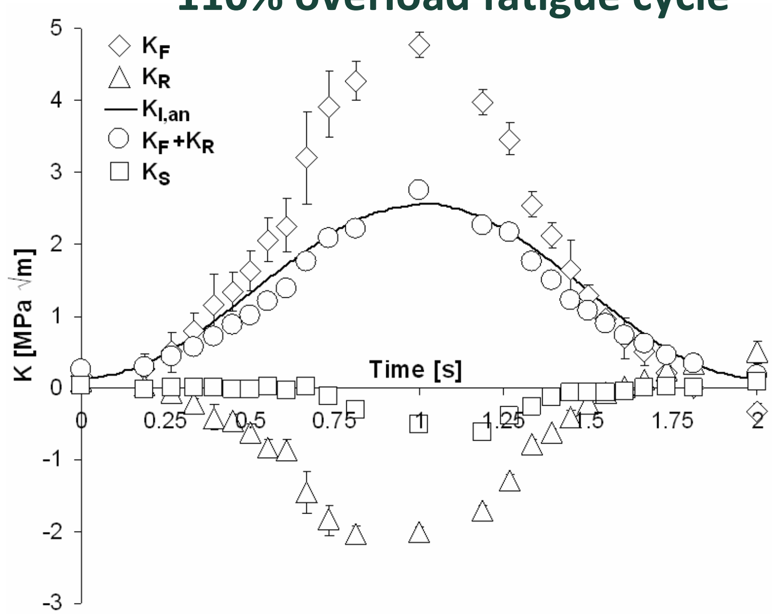


190N

## Constant amplitude fatigue cycle




## 110% overload fatigue cycle




Colombo, C., James, M.N., Patterson, E.A., Vergani, L., On crack tip shielding due to plasticity-induced closure during an overload, *FFEMS*, in press

# Conclusions

- **Robust and fast algorithm demonstrated on troublesome fringe pattern**
  - Extensive interaction between isochromatic & isoclinic parameters
  - Density and complex of fringes around the crack tip
- **Better than existing algorithms using a small number of images**
  - Previously tended to mask such areas which is not appropriate when data needed close to crack tip
  - More images not viable when analysis a propagating fatigue crack
- **Solution combines robust algorithms of Quiroga & Gonzálex-Cano[1] and fast algorithms of Siegmann et al[2]**
  - Novel application of cost function using quality map
  - Elimination of the minimization of the cost function at every pixel

1. Quiroga, J.A., Gonzalex-Cano, A., Separation of isoclinics and isochromatics from photoelastic data with a regularized phase-tracking technique, *Applied Optics*, 39:2931-2940, 2000.
2. Siegmann, P., Backman, D., Patterson, E.A. , A robust approach to demodulating and unwrapping phase-stepped data, *Experimental Mechanics*, 45(3):278-289, 2005.